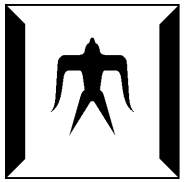
MATX (ver. 4.201)

Reference Manual



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Chapter 1

Introduction

MATX is a high-performance programming language for scientific and engineering numerical and symbolic computation [6], [7]. Since MATX integrates numerical analysis, matrix computation, and symbolic manipulation in an easy way to use, problem solutions are expressed almost exactly as they are written mathematically. Therefor, the users can solve a lot of computational problem in much shorter time than describing programs in any other programming languages, such as FORTRAN, PASCAL, C, and BASIC. Symbolic expressions such as polynomial and rational-function, polynomial matrix, and rational-function matrix are essential to the study of system theory and signal processing. MATX allows the definition and manipulation of these objects in a natural symbolic fashion.

The bigger a matrix, the more complicated to enter a matrix. If you make a slight mistake in entering a matrix, you must start from the beginning. The matrix-editor "**mated**" provides us with a very convenient user-interface to enter and revise matrices. Scientific and engineering data, which was created in MATX , are easily examined graphically in **XPLOT** [6]. It is a useful graphical drawing environment which runs under X Window System. The flexibility and strength of MATX is easily seen to be important for designers who want to combine the interactivity and the performance of optimized FORTRAN and C libraries.

1.1 Programming Language MATX

Many of the important ideas of matrix operation of MATX stem from the matrix manipulation language Matlab developed by Cleve Moler [8], [9]. Although it shares several characteristic features with Matlab, MATX is in no sense a dialect of it. Matlab is "typeless" language: the only data type is the Matrix, and operation of other kinds of objects is by special operators or function calls. MATX is equipped to recognize several primitive data type such as Integer, Real-Number, Complex-Number, String, Polynomial, Rational-Function, Matrix, Array, and List. Each class object is stored in an appropriate data structure and treated suitably. Matrix, Array, Polynomial, Rational-Function, and List don't require dimension.

It deals with symbolic formulae as well as numbers. It can solve polynomial equations and evaluate derivatives and integrals symbolically. MATX has inherited the idea of C language such as declaration of variables, control-flow, style of function, and structure of a program [5], [14]. It is written all in C language and the parser has been developed by using the well known parser generator "**yacc**" [13], [15].

MATX provides not only command-line *interpreter* **matx** whose interfaces are similar to the use of Matlab but also *compiler* **matc** which accepts the same syntax as that of matx and outputs portable C language code. We denote the package by MATX and distinguish it from the interpreter (matx). The users can extend the functionality of a program by realizing algorithms as functions in "mm-files" which correspond to "m-files" of Matlab. It is very easy to call your own C functions from MATX functions by linking C language routines to the output of matc. This allows us to utilize huge pre-exist C routines and speed up the rate of computation and promote the efficiency of memory usage.

1.2 Architecture of MATX

The architecture of MATX is shown in Fig. 1.1. At the moment, MATX provides three useful user-interface tools: interpreter "matx", compiler "matc", and matrix-editor "mated". They are built upon MaTX-Lib which is a collection of useful Class libraries such as **Complex-Number**, **Polynomial**, **Rational-Function**, **Matrix**, and **List**[6].

The library set MaTX-Lib can be utilized to make programs in C lan-

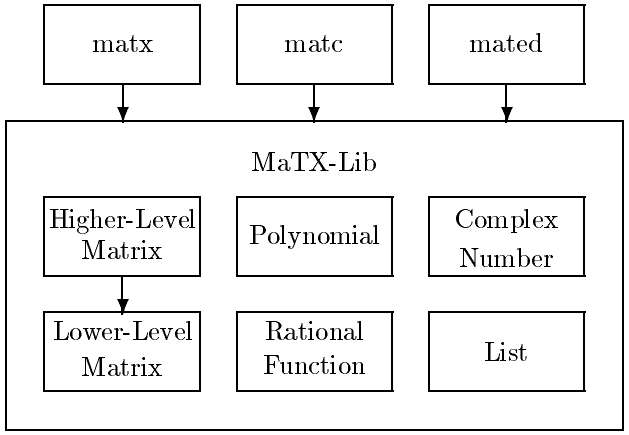

Figure 1.1: The Architecture of MATX

guage by itself. Matrix class library is composed of higher-level library and lower-level library. The lower-level library consists of nearly 300 functions, which provide fast access to the objects, but do little to make programming easier. It handles the operation of the objects and includes some optimizations that encourage efficient memory usage. The higher-level library is built upon lower-level library and consists of about 200 functions. The purpose of the higher-level library is to provide the supervisory layer that supports the user-interface facilities. It handles the memory allocation of object and calls suitable lower-level functions according to the attribute of the object.

Figure 1.2 indicates the internal process of MATX . A source file (`test.mm`) written in the MATX language is given to the lexical analyzer which recognizes the expressions in the input stream and partitions it into strings matching the MATX expressions. The Parser calls the lexical analyzer to pick up the partitioned basic items (tokens). These tokens are organized according to the MATX input structure rules; when one of these rules is recognized, the corresponding action for the rule is invoked. In each action, both the interpreter and the compiler defect type mismatches, inconsistent argument usage, unused or apparently uninitialized variables,

potential portability difficulties, and the like. In case of operations be-

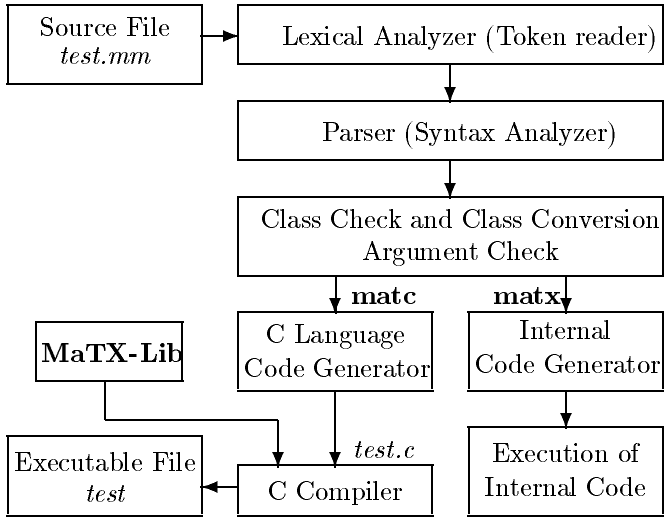

Figure 1.2: The Internal Process of MATX

tween distinct class objects, "Class Conversion" is done according to the "Automatic Class Conversion Rule" shown in Fig. 1.3. After the strict check to many aspects of a program, **matc** calls the "C Language Code Generator", while **matx** calls the "Internal Code Generator" and puts the generated code into execution.

1.3 Program Developing Process

Figure 1.4 and Fig. 1.5 show the program developing process in Matlab and MATX , respectively. In the program developing process in Matlab, the users usually realize algorithms in m-files (`test.m`) according to the requirements (`specs`). The m-files are tested several times until they satisfy the requirements. It is possible to call the user's C or Fortran subroutines (`ctest.mtex`), from Matlab which are produced from source code (`ctest.c`) by C or Fortran compiler. In the program developing process in

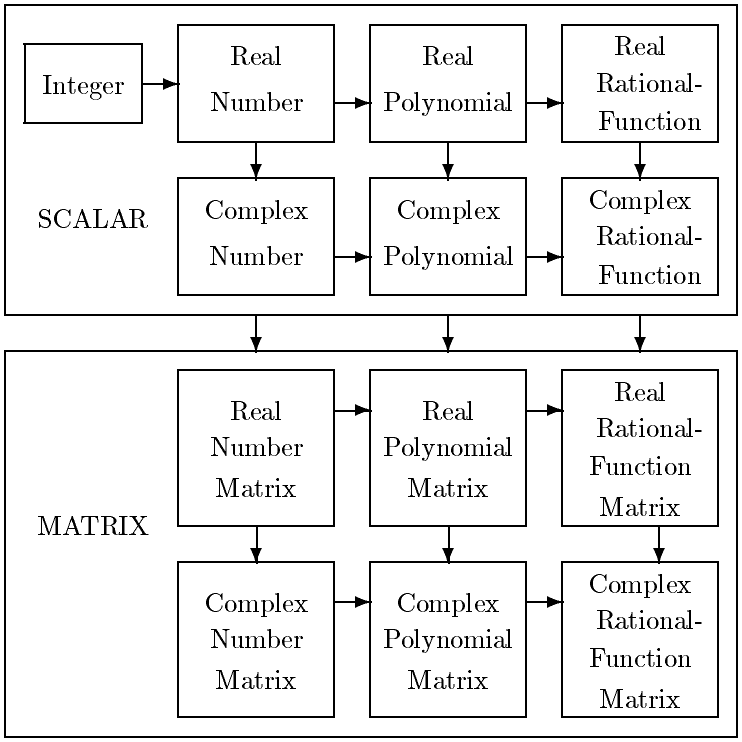

Figure 1.3: The Automatic Class Conversion Rule

MATX , the users ordinarily make several procedures (mm-files) using a favorite editor (`vi, emacs, or so`) according to the requirements (`specs`).

The interpreter (`matx`) enables us to test out the mm-files interactively file-by-file or line-by-line. The examined mm-files are compiled to portable C language files by the compiler (`matc`). It is possible to call your own C functions from MATX functions by linking C language files (`ctest.c`) to the output of matc (`test.c`). When all C language files are completed, they are compiled and linked up with the run-time library (`MaTX-Lib`) by C compiler (`cc`) to produce the desired executable program (`test`). Finally, the executable file is executed to check whether all requirements are

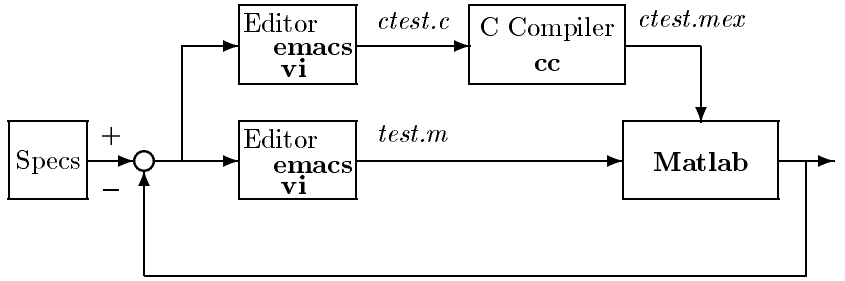

Figure 1.4: Program Developing Process in Matlab

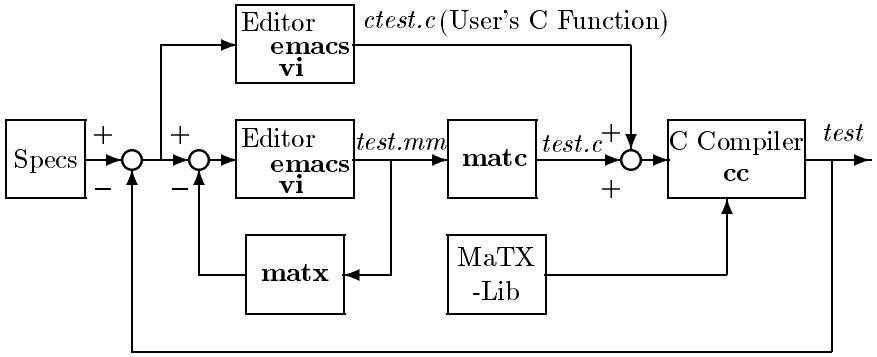

Figure 1.5: Program Developing Process in MATX

satisfied or not.

1.4 MATX Running Environment

At the moment, MATX is checked to run on the following machines under the operating system.

CRAY C916/12256 Cray UNICOS
HP 9000/755 HP-UX

Mips RC6280 UMIPS4.52C
SPARC Station20 SunOS4.1.3, SOLARIS 2.X
SPARC Station10 SunOS4.1.3, SOLARIS 2.X
JCC JS5 SOLARIS 1.1.1
SPARC Station classic SOLARIS 2.X
Sun4 SunOS4.0.3, SunOS4.1.X
Sun3 SunOS4.0.3, SunOS4.1.X
NEWS-830(SONY) NEWS-OS 2.0, 3.3
J3100 UNIX System V/386 3.0, PC-DOS 3.1
Gateway 2000 LINUX (1.0.9, 1.1.18)
Gateway 2000 DOS/V, (go32 + gcc, djgcc)
EPSON Note 386BSD, FreeBSD2.0
PC9801(NEC) MS-DOS 3.X, (go32 + gcc, djgcc)
X68000(SHARP) Human68k (gcc + libc)

Chapter 2

How To Make Programs and Execute Them

How to make programs and execute them depends on the machine and the operating system. In this manual, the programming developing process in UNIX operating system is illustrated.

2.1 How To Execute Programs

There are at least five ways to execute the program using the interpreter (matx) as follows.

1. Execute matx with source files as command line arguments. Source files are interpreted in turn. File extension "`mm`" can be omitted. It is convenient to run matx after adding a statement "`FunctionName();`" on the last line of the source file.

   [Example] Run matx after adding the statement "`main();`" on the last line of `sub3.mm`.

```
% emacs main.mm sub1.mm sub2.mm sub3.mm
% matx main.mm sub1.mm sub2.mm sub3.mm
```


2. If a minus "`-`" is specified at the end of the command line, matx becomes the interactive mode and shows a prompt "`MaTX (1)`" after it interprets all source files. Thereafter, enter the command "`FunctionName();`" to execute the function.

   [Example] In the interactive mode, enter the command "`main();`".

```
% emacs main.mm sub1.mm sub2.mm sub3.mm
% matx main.mm sub1.mm sub2.mm sub3.mm -

MaTX (1) main();
```

3. Run `matx` without arguments in order to invoke it as the interactive mode. Thereafter, read MM-FILES by the command `load`, and execute it. The default file extension is "`mm`".

   [Example] Add a statement "`main();`" at the end of `sub3.mm`, and read MM-FILES by the command `load`. beginfullbox beginexp*1em

   MaTX (1) load "main" "sub1" "sub2" "sub3" endexp* endfullbox

4. When the declared but not defined function is called, a MM-FILE which has the same name as the function is loaded. It is possible to call a function simply by the command "`FunctionName();`" after declaring it. The name of a function must be identical with the name of the MM-FILE for this facility.

   [Example] Declare the function and call it by its name.

```
% emacs main.mm sub1.mm sub2.mm sub3.mm
% matx

MaTX (1) void main();
MaTX (2) main();
```

5. Adding a line "`#! /usr/local/bin/matx`" at the top of MM-FILES and changing the attribute (executable permition) of the file, we can

execute MM-FILES like shell scripts and Perl script in UNIX. But the C-preprocessor facility is restrained.

[Example]

```
#! /usr/local/bin/matx -nocpp

Func void main()
{
  Matrix a, b, c;

  read a, b;
  c = a + b;
  prnit a, b, c;
}

main();
```

```
  % emacs test
  % chmod a+x test
  % ./test
```

2.2 How To Make Executable Programs

1. Create the source program in a file whose name extension is "`mm`", such as `test.mm`, using a favorite editor.

2. Compile it with the command "`matc test`", where the file-extension is optional on the command line. You can use the filename with another file-extension by specifying the option "`-nomm`." If you haven't botched anything, such as omitting a character or misspelling something, the compilation will proceed silently, and make a C language file called `test.c`. If you execute `matc` without arguments, it is invoked to become the interactive mode and shows a prompt "`matc (1)`". This mode is convenient for debugging the program. The source code of MATX is inserted in the C language code, if the option "-c" is specified.

3. Compile it with the command "`matcc test.c`", and make an executable file called `test`.

4. Running that by the command `test`. If any errors are found in the execution, error messages are displayed on the screen and the execution stops. More detail error information is written out to the file "`MaTX.log`".

[Example] Making an executable file `main` from four source files.

```
  % emacs main.mm sub1.mm sub2.mm sub3.mm
  % matc main.mm sub1.mm sub2.mm sub3.mm
  % matcc main.c sub1.c sub2.c sub3.c
  % main
```

2.3 C Language Interface

It is very easy to call C functions from MATX functions simply by linking C functions to the output codes from `matc`. The user must obey the following interface rules when he links his own C functions to MATX programs. Please refer to a library reference manual for a detail library explanation.

1. Include some header files at the beginning of each file. At the beginning of the function "`main()`", call four class initialization functions for Matrix class, Polynomial class, Rational-Function class, and List class, that is, "`MatInit()`", "`PolyInit()`", "`RatInit()`", and "`ListInit()`". Thereafter, initialize global variables of matrices, polynomials, rational functions, and lists, respectively.

```
#include <matrix.h>
#include <polynomial.h>
#include <rational.h>
#include <list.h>

Matrix a, b;     /* Global Matrix variable     */
Polynomial c, d; /* Global Polynomial variable */
Rational e, f;   /* Global Rational variable   */
List g, h;       /* Global List variable       */

void main(argc, argv)
    int  argc;
    char *argv[];
{
    /* Initialization of Classes */
    MatInit(); PolyInit();
    RatInit(), ListInit();

    /* Initialization of Global Variables */
    a = MAT_DEF,  b = MAT_DEF;
    c = POLY_DEF, d = POLY_DEF;
    e = RAT_DEF,  f = RAT_DEF;
    g = LIST_DEF, h = LIST_DEF;
    ......................
}
```


2. In order to release the memory area for local variables at the end of the function, write the following declarations and statements corresponding to the classes used in the function at the beginning and at the end of the function, respectively.

```
void foo()
{
    Matrix     mat_last  = MAT_DEF;
    Polynomial poly_last = POLY_DEF;
    Rational   rat_last  = RAT_DEF;
    List       list_last = LIST_DEF;

    ......................
    MatFrees(mat_last);
    PolyFrees(poly_last);
    RatFrees(rat_last);
    ListFrees(list_last);
}
```

3. Local variables of matrices, polynomials, rational functions, and lists should be declared and initialized at the one time.

```
void foo()
{
    ......................
    Matrix     aa = MAT_DEF,  bb = MAT_DEF;
    Polynomial cc = POLY_DEF, dd = POLY_DEF;
    Rational   ee = RAT_DEF,  ff = RAT_DEF;
    List       gg = LIST_DEF, hh = LIST_DEF;
    ......................
}
```

Chapter 3

Commands

3.1 Print Out To Display

All class objects can be outputted to standard-output by **print** command. Multiple objects can be specified by separating objects with commas.

```
print 1, 1.0, (1,2), "Hello", [[1 2][3 4]];
```

3.2 Read from Keyboard and Revision

All class objects can be inputted from standard-input by the **read** command. Multiple objects can be specified by separating objects with commas. If **read** command is used with matrices, the matrix-editor (mated) is invoked to read them.

```
read a, b, A, B;
```


3.3 Load MM-FILES (matx only)

The command `load` with a **MM-FILE** filename loads the file. Multiple files separated by commas can be specified at a time. Without the filename extension, "`mm`" is used. MM-FILES are searched through the following order:

1. The current directory
2. The directory which is defined by the environment variable `MATXINPUTS` (Multiple directories separated with colons "`:`" can be specified. For MS-DOS version, multiple directories are separated with semicolons "`;`")
3. The default MM-FILE directory, which is determined at the compile time.

The command `load`, by itself, loads the variables from the file "`MaTX.mm`".

```
load "filename1.mm", "filename2.mm";
load;
```

3.4 Input and Output of Data

MATX provides several ways to save and load data into and from files.

3.4.1 Input and Output of Matrix Data

To input matrix data from files and output matrix data to files, the **read** and **print** commands are used with the "**redirect**" symbols "`<<`" and "`>>`" like the UNIX shell. "`mat`" is recommended as the filename extension for matrix data files. Matrix data are saved in "**MAT Data Format**".

```
print A >> "filename.mat";
read A  << "filename.mat";
```

3.4.2 Input and Output of General Data

To output all types of data to a file the **print** command is used with the "**redirect**" symbols "`->`". In case of `matx`, "`mx`" is used as the filename extension if the extension is omitted. General data is saved in "**MX Data Format**".

```
print a, b, c, d -> "filename.mx";
```

To input all types of data from a file the **read** command is used with the "**redirect**" symbols "`<-`". In case of `matx`, "`mx`" is used as the filename extension if the extension is omitted. If the all variable names are omitted, all data in the file is loaded to the variables whose name are identical with that of the variables stored.

```
read a, b, c, d <- "filename.mx";
```

If any data are saved by the method except this one, the accuracy of the data may be deteriorated.

3.4.3 Save and Load of General Data

The command `save` with variables separated by commas and a filename saves the variables in the file. These variable can be loaded from the file by the command `load` Without the filename extension, "`mm`" is used. The command `save` with a filename saves all the variables in the file, and `save`, by itself, saves everything in a file named "`MaTX.mm`".

```
save M1, r1, P1, [[1 2][3 4]], "Hello", "file.mm";
save "file"; // All variables are saved in "file.mm"
save;        // All variables are saved in "MaTX.mm"
```

The command `load` loads the variables from a file which was save by the command `save`. Since the variables are loaded as global variables, the command `load` can not be used inside of functions. Without the filename extension, "`mm`" is used.


```
load "filename1.mm", "filename2.mm";
```

3.5 List of Variables and Functions (matx only)

The command `who` lists all the variables defined in the environment.

```
who;
```

The command `whos` shows the detail information of all the variables.

```
whos;
```

The command `what` lists all the functions defined in the environment.

```
what;
```

3.5.1 Clear Variables

Variables can be removed from the memory space by `clear` command. Multiple objects can be specified by separating variables with commas.

```
clear a, b, A, B;
```

3.6 Existence of Variables (matx only)

In order to check if a variable is defined or not, we can use the function `exist()` with the name of the variable. The function returns 1, 2, or 0, If the variable is defined as a local variable, if the variable is defined only as a global variable, if the variable is not defined.

```
i = exist("A");
if (i == 0)
  print "A dosen't exist.";
 else if (i == 1)
  print "A is a global variable.";
 else if (i == 2)
  print "A is a local variable.";
```

3.7 Getting and Changing Environment Variable

The function "`putenv()`" change or add value to environment, and the function "`getenv()`" return value for environment name.

```
putenv("DISPLAY=dazai:0.0");
disp = getenv("DISPLAY");
```

3.8 Generate amenu of choices

The function "`menu()`" displays the menu whose title is in the string variable (the first argument) and whose choices are string variables (the following arguments). `menu()` returns the value the user enters. We can specify the default item number of the menu as the second argument.

```
mode = menu("TITLE","item1","item2","item3","item4");

menu0 = "TITLE", "item1", "item2", "item3", "item4";
default_item = 2;
mode = menu(menu0, default_item);
```


3.9 Command of OS

The function "`system()`" send the arguments to operating system as a command.

```
system("xplot -silent A.mat");
```

3.10 Clear Screen (matx only)

The command `clear` clears the screen.

```
clear;
```

3.11 bell

The function `bell()` rings a bell.

```
bell();
```

3.12 Warning

The function `warning()` displays the text on the screen. The message is outputed to the **stderr**. If the function is called with an argument of a 0-length string, the warning is neglected.

```
warning("WARNING: Poles lie near the imaginary axis\n");
warning("Poles lie on the imaginary axis at (%f, %f)\n",
        Re(p), Im(p));
warning("");
```

3.13 Error

The function `error()` causes an error return to the keyboard and display the text on the screen. The message is outputed to the **stderr**. If the function is called with an argument of a 0-length string, the error is neglected, and if the function is called with no arguments, only the bell sounds and the execution is aborted.

```
error("ERROR: Zero division\n");
error("Dimension error A(%d, %d), B(%d, %d)\n",
       Rows(A), Cols(A), Rows(B), Cols(B));
error("");
error();
```

3.14 Help

Executing `help` gives a list of HELP topics, including the MM-file in the various libraries. `help topic` gives help on the specified topic. If the topic is not found in the help file, the help facility looks on the disk for an MM-file with the filename `topic.mm`. If it is found, the first comments on the top of the file are displayed. This allows you to use HELP on your own MM-files.

```
help;      // General help message is displayed
help eig;  // Help of builtin function eig is displayed
help Ode;  // Help of user function of Ode is displayed
```

3.15 Pause

The command `pause` stops the execution for the specified time.

```
pause;       // Stop until any key inputs
pause(5);    // Stop for file seconds
```


3.16 Quit

The command `quit` or `exit` exit from the interactive mode of **matx** and **matc**. The command `exit` returns the state.

```
quit;
exit;
exit -1;
```

Chapter 4

Matrix Editor

The bigger a matrix, the more complicated to enter a matrix. If you make a slight mistake in entering a matrix, you must start from the beginning. The matrix-editor "**mated**" provides us with a very convenient user-interface to enter and revise matrices. Even if the user enters wrong characters such as alphabetic characters and special characters, it will cause no problem, since mated takes in only suitable characters. The matrix-editor mated has the following features.

1. To handle any size of matrices and change their size arbitrarily.
2. Automatically scrolls if necessary.
3. To cut and paste any subportion of the matrix.
4. The key binding is similar to that of famous editors, **vi** and **emacs**.
5. To read and write matrices from and to files.
6. Don't require special window-system facilities.
7. To do a few matrix operations.

4.1 Edition of Real Matrices

```
 Name :  [ A ]
 Size :  [  50, 10 ]    Filename : A

            (  3)          (  4)          (  5)          (  6)
( 18)  -1.957854E-01  9.554574E-02 -1.957854E-01  9.554574E-02
( 19)  -1.953817E-01  9.751419E-02 -1.953817E-01  9.751419E-02
( 20)  -1.949779E-01  9.948264E-02 -1.949779E-01  9.948264E-02
( 21)  -1.945741E-01  1.014511E-01 -1.945741E-01  1.014511E-01
( 22)  -2.084542E-01  8.378548E-02 -2.084542E-01  8.378548E-02
( 23)  -2.080504E-01  8.575394E-02 -2.080504E-01  8.575394E-02
( 24)  -2.076466E-01  8.772239E-02 -2.076466E-01  8.772239E-02
( 25)  -2.072429E-01  8.969085E-02 -2.072429E-01  8.969085E-02
( 26)  -2.068391E-01  9.165930E-02 -2.068391E-01  9.165930E-02
( 27)  -2.064353E-01  9.362776E-02 -2.064353E-01  9.362776E-02
( 28)  -2.060315E-01  9.559621E-02 -2.060315E-01  9.559621E-02
( 29)  -2.056277E-01  9.756466E-02 -2.056277E-01  9.756466E-02
( 30)  -2.052239E-01  9.953312E-02 -2.052239E-01  9.953312E-02
( 31)  -2.048201E-01  1.015015E-01 -2.048201E-01  1.015015E-01
( 32)  -2.044164E-01  1.034700E-01 -2.044164E-01  1.034700E-01
```

Figure 4.1: Matrix Editor (Real Matrix) — mated

4.2 Edition of Complex Matrices

The command `C` transforms real matrices to complex matrices and gives back the complex matrix from real-part or imaginary-part matrix. (matx only) In editing the real-part or imaginary-part matrix, the size of the matrix can not be changed, the matrix can not be written out to a file and read from a file. Figure 4.2 shows the matrix editor which deals with a complex matrix, and Figure 4.3 and Figure 4.4 show the matrix editor which deal with the real-part matrix and imaginary-part matrix, respectively.

4.3 Key Bindings

The control key can be used before any alphabetic character and some symbols. For example, "^C" means to hold down the <CONTROL> key and type the "`C`" key at the same time. The <ESC> is a special key used

```
 Name :  [ B ]
 Size :  [  30, 10 ]   Filename : B

          (  3)-Real    (  3)-Imag    (  4)-Real    (  4)-Imag
( 12)   5.763537E-01  3.227756E-01  2.890290E-01  8.225142E-01
( 13)   2.218456E-01  1.713277E-01  5.039103E-01  1.292042E-01
( 14)   2.112349E-01  7.294220E-02  6.336816E-01  4.309787E-01
( 15)   8.716954E-01  9.988188E-02  6.955673E-01  7.323241E-01
( 16)   6.781542E-01  8.113680E-01  5.772123E-01  8.804363E-01
( 17)   6.445914E-01  9.994860E-01  2.568782E-02  8.759036E-03
( 18)   6.485325E-01  5.361216E-01  2.513150E-01  9.766774E-01
( 19)   2.967033E-01  7.648637E-01  6.800593E-01  4.797411E-01
( 20)   8.320193E-01  2.827639E-01  3.725870E-01  1.078266E-01
( 21)   8.935846E-01  1.882765E-01  9.587078E-01  9.792556E-01
( 22)   5.605379E-01  2.117229E-01  6.072429E-01  9.438229E-01
( 23)   8.295992E-01  1.647209E-01  5.731654E-01  7.800235E-01
( 24)   2.509799E-01  8.080054E-01  2.564957E-01  7.973468E-01
( 25)   3.942465E-01  4.115652E-01  5.600023E-01  5.074672E-01
( 26)   9.020740E-01  6.695332E-02  2.406348E-01  4.392214E-01
```

Figure 4.2: Matrix Editor (Complex Matrix)

to begin many commands as well. This key is pressed, and then released before typing the next character.
How to copy a submatrix to an area ? The first step is to define the submatrix. Set the mark at the beginning of the submatrix with <ESC>-<SPACE> **set mark** and move the cursor to the end of the submatrix. Now enter "`i`" **copy submatrix to kill matrix**. The kill matrix is now identical with the specified submatrix. Then move to the beginning of the destination area and enter "`o`" **yank back from kill matrix**.

4.4 Matrix Editor (mated)

The matrix editor `mated` can be used on the command line. It deals with MAT format matrix files.

```
% mated A.mat B.mat C.mat
```



```
 Name :  [ Re(B) ]
 Size :  [  30, 10 ]

          (  3)         (  4)         (  5)         (  6)
( 12)   5.763537E-01  2.890290E-01  3.210669E-01  2.613233E-01
( 13)   2.218456E-01  5.039103E-01  6.703021E-02  3.930634E-01
( 14)   2.112349E-01  6.336816E-01  5.356539E-02  7.834114E-01
( 15)   8.716954E-01  6.955673E-01  9.301896E-01  4.550905E-01
( 16)   6.781542E-01  5.772123E-01  2.731182E-01  9.353880E-01
( 17)   6.445914E-01  2.568782E-02  8.413805E-01  7.701880E-02
( 18)   6.485325E-01  2.513150E-01  2.286443E-01  2.509221E-01
( 19)   2.967033E-01  6.800593E-01  8.331472E-01  8.763075E-01
( 20)   8.320193E-01  3.725870E-01  7.570482E-01  1.092149E-01
( 21)   8.935846E-01  9.587078E-01  1.217351E-01  9.815443E-01
( 22)   5.605379E-01  6.072429E-01  8.164395E-01  4.465349E-01
( 23)   8.295992E-01  5.731654E-01  1.047806E-01  7.329764E-01
( 24)   2.509799E-01  2.564957E-01  3.381931E-01  3.882216E-01
( 25)   3.942465E-01  5.600023E-01  3.108211E-01  8.225768E-01
( 26)   9.020740E-01  2.406348E-01  3.731643E-01  7.522860E-01
```

Figure 4.3: Matrix Editor (Real Part Matrix)

```
Name :  [ Im(B) ]
Size :  [  30, 10 ]

          (  3)         (  4)         (  5)         (  6)
( 12)   3.227756E-01  8.225142E-01  1.384097E-02  9.237073E-01
( 13)   1.713277E-01  1.292042E-01  1.589902E-01  2.927582E-01
( 14)   7.294220E-02  4.309787E-01  7.498257E-01  1.519544E-01
( 15)   9.988188E-02  7.323241E-01  5.288297E-01  5.194745E-01
( 16)   8.113680E-01  8.804363E-01  1.127707E-03  1.070992E-01
( 17)   9.994860E-01  8.759036E-03  6.378548E-02  9.676939E-01
( 18)   5.361216E-01  9.766774E-01  2.334755E-01  4.185591E-01
( 19)   7.648637E-01  4.797411E-01  5.563773E-01  1.208822E-02
( 20)   2.827639E-01  1.078266E-01  2.987334E-01  5.287705E-01
( 21)   1.882765E-01  9.792556E-01  5.031973E-01  1.600592E-01
( 22)   2.117229E-01  9.438229E-01  6.220121E-01  2.136757E-01
( 23)   1.647209E-01  7.800235E-01  7.095231E-01  1.848203E-01
( 24)   8.080054E-01  7.973468E-01  9.864512E-01  6.998894E-01
( 25)   4.115652E-01  5.074672E-01  8.462741E-01  8.248976E-01
( 26)   6.695332E-02  4.392214E-01  8.449764E-01  3.010135E-01
```

Figure 4.4: Matrix Editor (Imaginary Part Matrix)

Table 4.1: Key Bindings of mated

Command	Emacs	Vi
Move forward by columns	^F	l
Move backward by columns	^B	h
Move backward by rows	^P	k
Move forward by rows	^N	j
Move to start of row	^A	^
Move to end of row	^E	$
Move to (first row, first column)	ESC <	g
Move to (last row, last column)	ESC >	G
Kill one row	^K r	d·r
Kill one column	^K c	d·c
Yank back from kill matrix	^Y	p
Redraw	^L	
Set mark	ESC SPACE	m
Copy submatrix to kill matrix	i	i
Yank back from kill matrix	o	o
Zero matrix		Z
Identity matrix		E
Real Part matrix		R
Imaginary Part matrix		I
Complex matrix		C
Transpose (Conjugate Transpose)		T
Read from a file	r	r
Write to a file	w	w
Exit from mated	q	q

Chapter 5

Graphing

Scientific and engineering data are examined graphically in MATX using "graphic functions" to create plots on the screen.

Table 5.1: Graphic Functions 1

Name	Arguments	Functions
plot(Y)	Y: Array	linear x-y plot (XPLOT)
plot(X, Y)	X: Array, Y: Array	linear x-y plot (XPLOT)
splot(X, Y)	X: Array, Y: Array	semi-log x-y plot (XPLOT) (x-axis logarithmic)
gplot(Y)	Y: Array	linear x-y plot (gnuplot)
gplot(X, Y)	X: Array, Y: Array	linear x-y plot (gnuplot)
gsplot(X, Y)	X: Array, Y: Array	semi-log x-y plot (gnuplot) (x-axis logarithmic)

If one uses a graphic function (`plot()`, `splot()`), the `XPLOT` is invoked automatically and the data are plotted on the 1D-canvas, 2D-canvas, or Semilog-Canvas. Therefore, the graphic functions can be used in X-Window System.

If one uses a graphic function (`gplot()`, `gsplot()`), the `gnuplot` is invoked automatically and the data are plotted on a window.

Before running the MATX , the user should set the name of terminal, which he or she uses, to the environmental variable `DISPLAY`:


```
% setenv DISPLAY hostname:0.0
```

Once a graph is on the screen, by using the mouse and popup menus, graph may be labeled, titled, or have scale lines drawn. It is also possible to chage the font of the labels and titles, to send a screen dump of the picture in the graph canvas to the printer, which is connected to the network, and to obtain the FIG code of the graph, which can be putted in the TEXtext.

Moreover the graph may be labeled, titled, or have grid lines drawn in:

Table 5.2: Graphic Functions 2

Name	Arguments	Functions
title(text)	text: String	graph title (XPLOT)
xlabel(text)	text: String	x-axis label (XPLOT)
ylabel(text)	text: String	y-axis label (XPLOT)
grid()		grid lines (XPLOT)

5.1 Basic Form

If `Y` is a vector, `plot(Y)` (`gplot(Y)`) produces a linear plot of the elements of `Y` versus the index of the elements of `Y`. For example, to plot the numbers {0., .48, .84, 1., .91, .6, .14}, enter them into a vector and call the function `plot()` (`gplot()`):

```
Y = [0. .48 .84 1. .91 .6 .14];
plot(Array(Y));
gplot(Array(Y));
```

This result in a graph on your screen. Notice that the data are auto-scaled and that X- and Y-axes are drawn. Once a graph is on the screen, by using the mouse and popup menus, graph may be labeled, titled, or have scale lines drawn.

If `X` and `Y` are vectors of the same length, the command `plot(X,Y)` (`gplot(X,Y)`) draws an x-y plot of the elements of `X` versus the elements of `Y`. For example,

```
t = [0.0:0.05:4*PI];
Y = sin(t);
plot(t, Y);
gplot(t, Y);
```

results in a sinusoidal plot of time, from `t = 0.0` to `t = 4*PI`.

5.2 Multiple Lines

There are two ways to plot multiple lines on a single graph. The first is to give `plot()` (`gplot()`) two arguments, as in `plot(X,Y)` (`gplot(X,Y)`), where either `X`, `Y`, or both, are matrices. Then

- If `Y` is a matrix, and `X` a vector, `plot(X,Y)` (`gplot(X,Y)`) successively plots the rows of `Y` versus the vector `X`, using a different line type for each.
- If `X` is a matrix, and `Y` a vector, then the above rules are applied except the family of lines from `X` are plotted versus vector `Y`.
- If `X` and `Y` are both matrices of the same size, `plot(X,Y)` (`gplot(X,Y)`) plots the rows of `X` versus the columns of `Y`.
- If no `X` is specified, as in `plot(Y)` (`gplot(Y)`), where `Y` is a matrix, then lines are plotted for each rows of `Y` versus the column index.

The second, and easier, way to plot multiple lines on a single graph is to use `plot` (`gplot`) with multiple arguments:

```
plot(X1, Y1, X2, Y2, ..., X5, Y5);
gplot(X1, Y1, X2, Y2, ..., X5, Y5);
```

The variables, `X1`, `Y1`, `X2`, `Y2`, etc. are pairs of vectors. Each x-y pair is graphed, generating multiple lines on the plot. Multiple arguments have

the benefit of allowing vectors of different lengths to be displayed on the same graph. As before, each pair uses a different line type.

5.3 Logarithmic Plots

The use of the function `splot()` (`gsplot()`) is identical to the use of `plot()` (`gplot()`). The function `splot()` (`gsplot()`) is a plot using semi-log scales. The x-axis is $\log_{10}$ while the y-axis is linear.

Chapter 6

Variable

In the compiler (matc), all variables must be declared before use. The variables in MATX are divided into two categories, that is, local variables and global variables. Each local variable in a routine comes into existence only when the function is called, and disappears when the function is exited. Because automatic variables come and go with function invocation, they do not retain their values from one call to the next, and must be explicitly set upon each entry.

As an alternative to local variables, it is possible to define variables which are external to all functions, that is, global variables with can be accessed by name by any function that cares to. Furthermore, because external variables remain in existence permanently, rather than appearing and disappearing as functions are called and exited, they retain their values even after the functions that set them are done. All global variables must be declared before the function "`main()`". Because external variables are globally accessible, they provide an alternative to function arguments and returned values for communicating data between functions. Any function may access an external variable by referring to it by name, if the name has been declared somehow. If a large number of variables must be shared among functions, external variables are more convenient and efficient than long argument lists. However this reasoning should be applied with some caution, for it can have a bad effect on program structure, and lead to programs with many data connections between functions.

6.1 Declaration of Variables

A declaration specifies a type, and is followed by a list of one or more variables of that type. Variables can be distributed among declarations in any fashion. The name of variables is a sequence of letters and digits; the first character must be a letter. The underscore "_" counts as a letter. Upper and lower case letters are different. Some identifiers are reserved for use as keywords, and may not be used otherwise. Variables may not be initialized in their declaration at the moment.

```
Integer    i1, i2, i3;   // Integer
Real       r1, r2, r3;   // Real Number
Complex    c1, c2, c3;   // Complex Number
String     S1, S2, S3;   // String
Polynomial P1, P2, P3;   // Polynomial
Rational   R1, R2, R3;   // Rational Function
Matrix     M1, M2, M3;   // Matrix
Array      A1, A2, A3;   // Array
List       L1, L2, L3;   // List
```

6.2 Scope Rule

The functions and external variables that make up a MATX program need not all be compiled at the same time; the source text of the program may be kept in several files, and previously compiled routines may be loaded from libraries.

The *scope* of a name is the part of the program over which the name is defined. For a local variable declared at the beginning of a function, the scope is the function in which the name is declared, and variables of the same name in different functions are unrelated. The same is true of the arguments of the function.

On the other hand, if a global variable is to be referred to before it is defined, or if it is used in a different source file from the one where the function "`main()`" is defined, then an `extern` declaration is mandatory.

It is important to distinguish between the *declaration* of an global variable and its *definition*. A declaration announces the properties of a variable

(its type, size, etc.); a definition also causes storage to be allocated. In case of the interpreter (matx), a declaration does not only announcement of the properties of a variable but also its storage.

When one use the interpreter (matx), in the outside of functions any kinds of data can be assigned to an arbitrary variable. In other words the class of the variable in the left-hand side may be different from that of the data in the right-hand side. It is a special exception for providing users with convenient session.

Chapter 7

Function

A MATX program, whatever its size, consists of one or more "functions" which specify the actual computing operations that are to be done. MATX function are similar to the functions of C program. Normally you are at liberty to give functions whatever names you like, but "`main`" is a special name — your program begins executing at the beginning of `main` when you make the executable program using compiler (matc). This means that every program *must* have a `main` somewhere. However the program can begin executing wherever in the interpreter (matx).

The type of the function "`main()`" is `Integer` or `void`. When the type of "`main()`" is `Integer`, it can return a value to the caller as the other functions. In this case the caller is the environment where the program is executed. The return value 0 means normal exit and the other value means error or abnormal exit.

MATX functions may be used recursively; that is, a function may call *itself* either directly or indirectly. The local variables of the function are generated as the function is called. The definition of functions can not be nested. A program may reside on one or more source files in any convenient way; the source files may be compiled separately and loaded together, along with previously compiled functions from libraries.

7.1 Declaration of Functions

After the declaration of a function, it is called wherever. The name of functions is a sequence of letters and digits; the first character must be a letter. The underscore "_" counts as a letter. Upper and lower case letters are different. Some identifiers are reserved for use as keywords, and may not be used otherwise.

```
Integer    func_int1(), func_int2();
Real       func_real1(), func_real2();
Complex    func_cmp1(), func_cmp2();
String     func_str1(), func_str2();
Polynomial func_poly1(), func_poly2();
Rational   func_rat1(), func_rat2();
Matrix     func_mat1(), func_mat2();
Array      func_arr1(), func_arr2();
List       func_list1(), func_list2();
void       func_void1(), func_void2();
```

7.2 Definition of Functions

MATX functions are almost same as K&R C language functions except that it starts with a functions descriptor `Func`. A class name follows `Func` and `void` is used for the functions which don't return any values. Each statement in function is separated by semi colons. There is at least one `return` statement in a function which is not a `void` function.

```
Func ClassName FunctionName(arg1, arg2, arg3, ..., argN)
ClassName1 arg1, arg2;
ClassName2 arg3;
..............
ClassNameM argN;
{
    Declaration of variables.

    Statements.

    return;       // void function
    return value; // Otherwise
}
```

7.3 Multiple-Output Functions

In order to implement the multiple-output functions, we introduce the "List Class". Multiple-output function calls have the form like this

```
Integer a, d;
Complex b, f;
Matrix c, e;
List fun();
{a, b, c} = fun(d, e, f);
```

"`a, b, c`" are returned values and "`d, e, f`" are arguments of the function. Function "`fun()`" returns multiple output values as a list which contains them. The number of left-hand-side arguments must be less than or equal to that of the elements of the returned list. If the number of left-hand-side arguments is less than that of the elements of the list, the only top elements of the list which correspond to the left-hand-side arguments are used and the rest are discarded.

7.4 Variable Number of Arguments

In order to implement functions with variable number of arguments, we avail ourselves "List Class". As an example of a function with variable number of arguments, we observe the following function which calculates the sum of the argument matrices and returns it. Here, we assume that all matrices have the same size.

```
Func Matrix sum_mat(args)
  List args;
{
  Integer i, argn;
  Matrix ss;

  argn = length(args);
  ss = args(1, Matrix);
  for (i = 2; i <= argn; i++) {
    ss = ss + args(i, Matrix);
  }
  return ss;
}
```

In order to call the function with variable number of arguments, we make a list from the arguments and send it to the function. The function "`sum_mat()`" is called like

```
a = [[1 0][0 1]]; b = [[1 2][3 4]];
c = [[4 3][2 1]]; d = [[0 1][1 0]];
print sum_mat(a, b, c, d);
```

7.5 Calling Functions (matx)

A function is call by its name and the parentheses following the function name surround the argument list. The parentheses can not be omitted even if there is no arguments. When the function which has been declared

but not defined, a MM-FILE with the same name as the function is loaded. MM-FILES are searched as the following order: the current directory, the directory which is defined in the environmental variable `MATXINPUTS`, and the default MM-FILE directory. The environmental variable can contain multiple pathes separated by colons.

7.6 Arguments of Functions

One method of communicating data between functions is by arguments. The parentheses following the function name surround the argument list.

When an integer, a real number, a string, or a complex number appears as an argument to a function, they are passed by value, that is, the called function receives a private, temporary copy of each argument. This means that the function cannot affect the original argument in the calling function. On the other hand, when a matrix, a polynomial, a rational function, or a list appears as an argument to a function, the reference to them is passed; elements are not copied. The function can alter elements of the variable.

```
Func void main()
{
    Matrix a;
    void foo();

    a = [1];
    print a;       // a = [1]
    foo(a);
    print a;       // a = [2]
}

Func void foo(b)
    Matrix b;
{
    print b;       // b = [1]
    b = [2];
    print b;       // b = [2]
}
```


7.7 Return Statement

The `return` statement is the mechanism for returning a value from the called function to its caller. Any expression can follow `return`. The calling function is free to ignore the returned value if it wishes. There must be one expression after `return`; in case of value-return-functions. Furthermore, there must be no expression after `return`; in case of `void` function.

```
return;       // void function
return expr;  // Otherwise
```

7.8 Reference to Functions

In MATX a function is not a variable, but it is possible to pass its reference to functions. The reference to a function behaves like its name. However there is no description corresponding to a pointer of function in C.

```
Func void main()
{
  Matrix a;
  void foo(), bar();

  a = [[1 2][3 4]];
  foo(bar, a);
}

Func void foo(afo, a)
  void afo();
  Matrix a;
{
  afo(a);
}

Func void bar(a)
  Matrix a;
{
  print a;
}
```

7.9 Inline Functions(matc)

Once an expression is described with some variables, this expression is reusable with other parameters like a macro since the compiler matc holds its definition as well as the value in the effective scope.

```
r1 = 1.0;
M1 = [r1 2.0];

r1 = 2.0;
M2 = @M1;  // M2 = [2.0 2.0];
```

7.10 Command-Line Arguments

In environments that support MATX , there is a way to pass command-line arguments or parameters to a program when it begins executing. When "`main()`" is called to begin execution, it is called with two arguments. The first is the number of command-line arguments the program was invoked with; the second is a list that contain the arguments, one per string. The first string is the name of command. In order to pass an integer or a real number, the class conversion functions are exploited. For example,

```
Func void main(argc, argv)
    Integer argc;
    List argv;
{
    Integer a, b;

    print "Command Name is", argv(1, String), ".\n";
    a = Integer(argv(2, String));
    b = Integer(argv(3, String));
    print a + b;
}
```

prints out the command name and the sum of the first and second command-line argument integers.

Chapter 8

Control Flow

The control flow of MATX is similar to that of C language.

8.1 If-Else

The `if-else` statement is used to make decisions. Formally, the syntax is

```
if (expr) {
    statement-1;
} else {
    statement-2;
}
```

where the `else` part is optional. This makes an ambiguity when an `else` part is omitted from a nested `if` sequence. Therefore the braces "{" and "}" can not be omitted. If there is an `else` part, `else` part is supposed to begin at the last line of `if` part, since `matx` interprets line by line. The `expr` is evaluated; if it is "true" ,that is, if `expr` has a non-zero value, `statement-1` is done. If it is "false" (`expr` is zero) and if there is an `else` part, `statement-2` is executed instead.

8.2 Loops — While and For

In

```
while (expr) {
    statements;
}
```

the `expr` is evaluated. If it is non-zero, `statement` is executed and `expr` is re-evaluated. This cycle continues until `expr` becomes zero, at which point execution resumes after `statement`. The `for` statement

```
for (expr1; expr2; expr3) {
    statements;
}
```

is equivalent to the following.

```
expr1;
while (expr2) {
    statements;
    expr3;
}
```

The three elements in `for` statement are expressions syntacticly. In general `expr1` and `expr3` are assignment or function-call, and `expr2` is a relational expression. All three elements may be omitted, but semicolons are not. If the checking part (`expr2`) is omitted, it is treated as true forever. Therefore

```
for (;;) {
    statements;
}
```

is a "infinite" loop, and the loop exits are controlled by other methods (such as `break` and `return`).

8.3 Loops — Do-while

The `do-while` loop tests at the bottom after making each pass through the loop body; the body is always executed at least once. The syntax is

```
do {
    statements;
} while (exp) ;
```

The `statements` are executed, then `expr` is evaluated. If it is true, `statements` are executed again, and so on. If the expression becomes false, the loop terminates.

8.4 Switch

The `switch` statement is a spcial multi-way decision maker that tests whether an expression matches onee of a number of constant values, and branches accordingly.

```
switch (exp) {
 case int_1: statements;
 case int_2: statements;
 ......................
 case int_n: statements;
 default: statements;
}
```

The `switch` evaluates the integer expression in parentheses and compares its value to all the cases. Each case must be labeled by an integer constant. If a case matches the expression value, execution starts at that case. The case labeled `default` is executed if none of the other cases if satisfied. The `break` statement causes an immediate exit from the `switch`.

8.5 Break and Continue

It is sometimes convenient to be able to control loop exits other than by testing at the top or bottom.


```
break;
```

The `break` statement provides an early exit from `for` and `while`. A `break` statement causes the innermost enclosing loop to be exited immediately.

```
continue;
```

The continue statement is related to `break`, but less often used; it causes the *next iteration* of the enclosing loop (`for` and `while`) to begin. In the `while` loop, this means that the re-initialization step.

Chapter 9

Miscellaneous

9.1 Class Conversion Functions

Although the operations between distinct class objects invoke the automatic class conversions, some operations require explicit class conversions. The class conversion functions can be used for this. Their name is identical with the name of the destination class. For example, the statements

```
A = [[1 2][3 4]];  B = [[5 6][7 8]];
C = A .* B;
D = Array(A) * Array(B);
```

make matrix `C` equal to matrix `D`. The following statements uses the function "`Real()`" to converts a `1`×`1` real-matrix to a real number.

```
A = Real([1 2 3] * [4 5 6]') * [[1 2][3 4]];
```

The function "`String()`" in the statements,


```
A = [[1 2][3 4]];
for (i = 1; i <= 10; i++) {
  print A * i >> "A-" + String(i) + ".mat";
}
```

is used to convert an integer to a string.
MATX provides the following class conversion functions.

```
s = String(Integer)           Integer to string.
s = String(Real Number)       Real number to string.

i1 = Integer(String)          String to integer.
i1 = Integer(Real Number)     Integer nearest to real number.
i1 = Integer(Real Matrix)     Integer part of (1,1).
i1 = Integer(Complex Matrix)  Integer part of Re((1,1)).
i1 = Integer(Real Array)      Integer part of (1,1).
i1 = Integer(Complex Array)   Integer part of Re((1,1)).

r1 = Real(String)             String to real number.
r1 = Real(Real Matrix)        (1,1)-element.
r1 = Real(Complex Matrix)     Real-part of (1,1).
r1 = Real(Real Array)         (1,1)-element.
r1 = Real(Complex Array)      Real-part of (1,1).
r1 = Real(Complex Number)     Real part.

c1 = Complex(Real Matrix)     Re(c1) = (1,1).
c1 = Complex(Complex Matrix)  (1,1).
c1 = Complex(Real Array)      Re(c1) = (1,1).
c1 = Complex(Complex Array)   (1,1).
c1 = Complex(Real Number)     Complex number.

P1 = Polynomial(Real Number)   Coefficients are identcal
                               with the elements of the row
                               row vector in ascending order.
P1 = Polynomial(Real Number)     Constant real polynomial.
CP1 = Polynomial(Complex Number) Constant complex polynomial.
```

```
CP1 = CoPolynomial(Real Polynomial P) Re(CP1) = P, Im(CP1) = 0

R1 = Rational(Real Polynomial)  Numerator is the polynomial,
                                Denominator is 1.
R1 = Rational(Real Matrix M1)   Numerator is Polynomial(M1),
                                Denominator is 1.
R1 = Rational(Real Number r)    Numerator is Polynomial(r1),
                                Denominator is 1.
CR1 = Rational(Complex Number c)
                                Numerator is Polynomial(c1),
                                Denominator is 1.

CR1 = CoRational(Real Rational R) Re(CR1) = R, Im(CR1) = 0

M1 = Matrix(Array)              Array to Matrix.
M1 = Matrix(Polynomial)         Coefficients are identical
                                with the elements of the row
                                vector in ascending order.
M1 = Matrix(Real Number r)      1x1 real matrix.
CM1 = Matrix(Complex Number c)  (1,1) complex matrix.

CM1 = CoMatrix(Real Matrix|Array)       Re(CM1) = M, Im(CM1) = 0
PM1 = PoMatrix(Real Matrix|Array)       Const. term matrix is M
CPM1 = PoMatrix(Compex Matrix|Array)    Const. term matrix is M
RM1 = RaMatrix(Real Matrix|Array)       Const. term matrix of
                                        numerator matrix is M
CRM1 = RaMatrix(Complex Matrix|Array)   Const. term matrix of
                                        numerator matrix is M
RM1 = RaMatrix(Polynomial Matrix|Array) Numerator matrix is M

A1 = Array(Matrix)              Matrix to Array.
A1 = Array(Polynomial)          Coefficients are identical
                                with the elements of the row
                                vector in ascending order.
A1 = Array(Real Number r)       1x1 real array.
CA1 = Array(Complex Number c)   1x1 complex array.
```



```
CM1 = CoArray(Real Array|Matrix)       Re(CM1) = M, Im(CM1) = 0
PM1 = PoArray(Real Array|Matrix)       Const. term array is M
CPM1 = PoArray(Compex Array|Matrix)    Const. term array is M
RM1 = RaArray(Real Array|Matrix)       Const. term array of
                                       numerator array is M
CRM1 = RaArray(Complex Array|Matrix)   Const. term array of
                                       numerator array is M
RM1 = RaArray(Polynomial Array|Matrix) Numerator array is M
```

9.2 An Expression Across Lines (Command-line)

An operator or a comma "," at the end of a line means that the expression continues to the next line. However, a new line can not be started among inequalities and complex-value expressions.

```
a = 3 * 4 + 5 * 6 +
    7 * 8 + 9 * 10;

F = H_infty(A11, A12, A21, A22,
            B1, B2, C1, C2);
```

9.3 Assignment Expressions

MATX assignment has a value and assignment associate right to left. However, the assignment for extracting some elements from a list, the assignment for setting elements of matrices, the assignment for setting coefficients of polynomials, and the assignment for setting elements of lists, are not expressions but statements. For example,

```
Matrix A;
Polynomial B;
List C;

A = Z(2);
A(1,1) = 1.0;           /* Statement */
A(1,2) = [[1 2][3 4]];  /* Statement */

B = Polynomial(Z(1,4));
B(1) = 3.0;             /* Statement */

C = makelist(3);
C(1) = 1;               /* Statement */
```

And

```
Matrix A, B, C;

A = B = C = Z(2,3);
```

are correct, but

```
Matrix A, B, C;
List LL;

LL = {1, (1,2), [1 2]};
{A, B, C} = {A, B} = LL;
```

are wrong.

9.4 "ans": answer when expression is not assigned

When an expression is not assigned to anything else, its value is assigned to the variable "ans". For example

```
 MaTX (1) 1 + 2 + 3
ans = 6
 MaTX (2) ans*ans
ans = 36
 MaTX (3) sqrt(ans)
ans = 6
```

9.5 Complex-Valued Expressions

Complex valued expressions are allowed in all operations in MATX . They are entered as a couple of real valued expressions separated by a comma "," inside parentheses, "(" and ")". For example, entering the statement

```
a = (3,4);
```

results in

```
a = (3.00000000E+00,  4.00000000E+00)
```

There are at least two convenient ways to enter complex matrices. They are illustrated by the statements

```
A = ([[1 2][3 4]], [[5 6][7 8]]);
```

and

```
A = [[(1,5), (2,6)][(3,7), (4,8)]];
```

which produce the same result. The former statement uses a pair of real matrices as the real-part and the imaginary-part of the matrix. The latter

one uses four complex numbers as elements of the matrix. And entering the statements

```
x = $;
A = 3*x^2 + 4*x + 5;
B = 6*x^2 + 7*x + 8;
C = (A, B);
```

results in the output

```
C = (3,6) s^2 + (4,7) s + (5,8)
```

Real-part and imaginary-part of a complex value can be extracted by using the functions "`Re()`" and "`Im()`", respectively.

9.6 Comment

Any characters between "`/*`" and "`*/`", and any characters after "`//`" to the end of the line are ignored by the compiler and the interpreter; they may be used freely to make a program easier to understand. Comments may appear anywhere a blank or newline can. Comments between "`/*`" and "`*/`" can not be nested.

9.7 Preprocessor

MATX provides certain language extensions by means of the C preprocessor. The `#define` capability which we have used is the most common of these extensions; another is the ability to include the contents of other files. The default C preprocessor is `cpp`, and another preprocessor can be used by specifying its name with the environmental variable `MATXCPP`.

```
setenv MATXCPP /usr/local/lib/gcc-lib/cpp
```

If "`-nocpp`" flag is specified on the command line, no preprocessor is used.

9.7.1 File Inclusion

In order to facilitate handling collections of `#define`'s and declarations MATX provides a file inclusion feature. Any line that looks line

```
#include "filename.mm"
```

is replaced by the contents of the file `filename.mm`. `#include` is the preferred way to tie the declarations together for a large program. The named file is searched for first in the directory of the original source file, and then in a sequence of standard places (/usr/local/MaTX/inputs). Alternatively, a control line of the form

```
#include <filename.mm>
```

searches only the standard places, and not the directory of the source file.

9.7.2 Macro Substitution

A definition of the from

```
#define     YES    1
```

calls for a macro substitution of the simplest kind — replacing a name by a string of characters. Normally the replacement text is the rest of the line; a long definition may be continued by placing a \ at the end of the line to be continued. The "scope" of a name defined with `#define` is from its point of definition to the end of the source file.

It is also possible to define macros with arguments, so the replacement text depends on the way the macro is called. For example, define a macro called `ABCD` like this:

```
#define ABCD(S)  S(1,Matrix), S(2,Matrix), S(3,Matrix), S(4,Matrix)
```

Now the line

```
S = tf2ss((s+1)/(s^2+3*s+4));
bodeplot(ABCD(S))
```

will be replace by the line

```
S = tf2ss((s+1)/(s^2+3*s+4));
bodeplot(S(1,Matrix), S(2,Matrix), S(3,Matrix), S(4,Matrix));
```

This provides a "ABCD function" that expands into in-line code rather than a function call.

9.7.3 Conditional compilation

A compiler (interpreter) control line of the form

```
#if constant-expression
```

checks whether the constant expression evaluates to non-zero. A control line of the from

```
#ifdef identifier
```

checks whether the identifier is currently defined in the preprocessor, that is, whether it has been the subject of a `#define` control line. A control line of the form

```
#ifndef identifier
```

checks whether the identifier is currently undefined in the preprocessor.

All the forms above are followed by an arbitrary number of lines, possibly containing a control line

```
#else
```

and then by a control line

```
#endif
```

If the checked condition is true then any lines between `#else` and `#endif` are ignored. If the checked condition is false then any lines between the test and an `#else` or `#endif` are ignored.

9.8 Startup File (matx)

When the `matx` or `matc` is invoked, the startup file is searched and loaded, if it exits, as follows

1. "`.matxrc`" in the current directory. (For MS-DOS version, "`_matxrc`")
2. "`.matxrc`" in the home directory (The directory which is registered in the environmental variable `HOME`. (For MS-DOS version, "`_matxrc`")
3. (a) "`MaTXRC.mm`" in the current directory.

   (b) "`MaTXRC.mm`" in the directory which is defined by the environmental variable "`MATXINPUTS`"
   (Multiple directories separated with colons "`:`" can be specified. For MS-DOS version, multiple directories are separated with semicolons "`;`")
   (c) "`MaTXRC.mm`" in the default `MM-FILE` directory, which is determined at the compile time.

where, only one startup file is loaded.

9.9 Error Message

If any erros are detected by matx or matc, an error message is displayed on the screen and more detail error information is written out to the logfile "`MaTX.log`". There are two kinds of error messages. One is outputed by matx or matc, and the other is outputed by the Run-Time libraries. The style of error message from matx or matc is like

```
(1) "Filename",   LineNumber, VariableName, Reason
(2)             FunctionName, VariableName, Reason
```

and the style of error message from the Run-Time libraries is like

```
FunctionName: [Reason] in VariableName
```

9.10 Bugs

Chapter 10

Integer and Real Number

Declaration:

```
Integer a, b, c;
Real aa, bb, cc;
```

Example:

```
0, 3, -4                    (Integer)
0., 0.0, 3., 3.0, -4. -4.0  (Real-Number)
3E4, 3.E4., 3e4, 3.e4.      (Real-Number)
```

MATX distinguish between **Integer** and **Real Number** 1is an integer and `1.0` is a real number. In MATX , as many other languages, integer division truncates, so any fractional part is discarded. All real number arithmetic in MATX is done in double precision.

MATX provides five special constants `PI, E, GAMMA, DEG, PHI.` as primitives.


```
PI    =  3.14159265358979323846; /* Circumf./Diameter */
E     =  2.71828182845904523536; /* Napier's Number   */
GAMMA =  0.57721566490153286060; /* Euler's Constant  */
DEG   = 57.29577951308232087680; /* Degree/Radian     */
PHI   =  1.61803398874989484820; /* Golden Ratio      */
PID   =  ??????;                 /* Process ID        */
EPS   =  pow(2.0, -54.0);        /* Machine Epsilon   */
Inf   =  Infinity;               /* Infinity          */
                                 /* Ex. 1.0/0.0       */
                                 /* positive infinity */
NaN   =  NaN;                    /* Not a Number      */
                                 /* Ex. 0.0/0.0       */
```

10.1 Comparison of Integers and Real Numbers

We can compare two integers (and real numbers) by using relational operators `>, >=, <, <=, ==, !=`, and the result is an integer 1 (true) or 0 (false).

```
int1 > int2     int1 is greater than int2
int1 >= int2    int1 is greater than or equal to int2
int1 < int2     int1 is less than int2
int1 <= int2    int1 is less than or equal to int2
int1 == int2    int1 is equal to int2
int1 != int2    int1 is not equal to int2
re-n1 > re-n2   re-n1 is greater than re-n2
re-n1 >= re-n2  re-n1 is greater than or equal to re-n2
re-n1 < re-n2   re-n1 is less than re-n2
re-n1 <= re-n2  re-n1 is less than or equal to re-n2
re-n1 == re-n2  re-n1 is equal to re-n2
re-n1 != re-n2  re-n1 is not equal to re-n2
```

10.2 Real Number Function

MATX provides the following real number functions. The arguments of the function must be real number. The class conversion function "`Real()`" is used in order to use any integers as the arguments.

```
Integer rem(i1, i2)       Remainder (i1 / i2)
Integer max(i1, i2)       Maximum value
Integer min(i1, i2)       Minimum value
Integer sgn(i1)           Sign function (i1 > 0, 1;
                                         i1 = 0, 0;
                                         i1 < 0, -1)
Real rem(r1, r2)      Remainder (r1 - r2 * fix(r1/r2))
Real max(r1, r2);     Maximum value
Real min(r1, r2);     Minimum value
Real sin(Real r1);    Sine function
Real sinh(Real r1);   Hyperbolic sine function
Real asin(Real r1);   Arcsine function
Real asinh(Real r1);  Arc-hyperbolic sine function
Real cos(Real r1);    Cosine function
Real cosh(Real r1);   Hyperbolic cosine function
Real acos(Real r1);   Arccosine function
Real acosh(Real r1);  Arc-hyperbolic cosine function
Real tan(Real r1);    Tangent function
Real tanh(Real r1);   Hyperbolic tangent function
Real atan(Real r1);   Arctangent function (1)
Real atanh(Real r1);  Arc-hyperbolic tangent function
Real atan2(Real y1, Real x1); Arctangent function (2)
Real abs(Real r1);    Absolute value
Real exp(Real r1);    Exponential function
Real sqrt(Real r1);   Square root function
Real log(Real r1);    Natural logarithm function
Real log10(Real r1);  Common logarithm function
Real rand();          Uniformly distributed number
Real rand("seed");    Seed of u-distributed number
Real rand("seed", Integer val);
                      Set the seed of u-dist. number
Real randn();         Normaly distributed number
```



```
Real randn("seed");   Seed of n-distributed number
Real randn("seed", Integer val);
                      Set the seed of n-dist. number
Real fact(Real r1);   Factorial function
Real ceil(Real r1);   Rounding function
Real floor(Real r1);  Rounding function
Real fix(Real r1);    Rounding function towards zero
Real round(Real r1);  Rounding function to
                      the nearest integers
Real sgn(Real r1);    Sign function (r1 > 0.0, 1.0;
                                     r1 = 0.0, 0.0;
                                     r1 < 0.0, -1.0)
Integer finite(Real r1);  returns 1 if r1 are finite or zero,
                          and 0 if r1 are infinite or NaN
Integer isnan(Real r1);   returns 1 if r1 are NaN,
                          and 0 if r1 are not.
```

Chapter 11

Complex Number

Declaration:

```
Complex a, b, c;
```

Example:

```
(1, 2), (3.0, 4.0), (0, 1)
```

11.1 How To Describe Complex Numbers

Complex numbers are entered as a couple of real numbers separated by a comma "," inside parentheses, "(" and ")". Parentheses and a comma is call a **complex operator**. The real-part or imaginary-part can be omitted by using "*" or ":" instead. If the element is an integer, it is converted to a real number.

```
c1 = (r1,r2) /* c1 = r1 + r2*i; (r1, r2: real numbers) */
c1 = (r1, *) /* c1 = r1 + 0*i;  (r1: real number)      */
c1 = (r1, :) /* c1 = r1 + 0*i;  (r1: real number)      */
c1 = (*, r1) /* c1 = 0  + ri*i; (r1: real number)      */
c1 = (:, r1) /* c1 = 0  + ri*i; (r1: real number)      */
```


Real-part and imaginary part of a complex number can be extracted by using the functions "Re()" and "Im()", respectively.

```
r1 = Re((r1, r2));  /* Real part      */
r2 = Im((r1, r2));  /* Imaginary part */
```

Imaginary unit can be defined at least by the following methods.

```
i = (0, 1);
i = sqrt(-1);
```

11.2 Comparison of complex numbers

We can compare two complex numbers by using relational operators ==, !=, and the result is an integer 1 (true) or 0 (false).

```
comp1 == comp2  comp1 is equal to comp2
comp1 != comp2  comp1 is not equal to comp2
```

11.3 Complex Number Function

MaTX provides the following Complex-number functions.

```
Real    abs(Complex c1);    Absolute value
Real    arg(Complex c1);    Argument
Complex max(c1, c2);        Maximum value
Complex min(c1, c2);        Minimum value
Complex sin(Complex c1);    Sine function
Complex sinh(Complex c1);   Hyperbolic sine function
Complex asin(Complex c1);   Arcsine function
Complex asinh(Complex c1);  Arc-hyperbolic sine function
Complex cos(Complex c1);    Cosine function
Complex cosh(Complex c1);   Hyperbolic cosine function
Complex acos(Complex c1);   Arccosine function
```

```
Complex acosh(Complex c1);  Arc-hyperbolic cosine function
Complex tan(Complex c1);    Tangent function
Complex tanh(Complex c1);   Hyperbolic tangent function
Complex atan(Complex c1);   Arctangent function
Complex atanh(Complex c1);  Arc-hyperbolic tangent function
Complex log(Complex c1);    Natural logarithm function
Complex log10(Complex c1);  Common logarithm function
Complex exp(Complex c1);    Exponential function
Complex conj(Complex c1);   Complex conjugate function
Complex sqrt(Complex c1);   Square root function
Complex sgn(Complex c1);    Sign function (c1 / abs(c1))
Integer finite(Complex c1); returns 1 if c1 are finite or zero,
                            and 0 if c1 are infinite or NaN
Integer isnan(Complex c1);  returns 1 if c1 are NaN,
                            and 0 if c1 are not.
```

Chapter 12

String

Declaration:

```
String a, b, c;
```

Example:

```
"FileName", "File\tName\n"
"Hello" + " " + "world" /* "Hello world" */
```

A string constant is a sequence of zero or more characters surrounded by double quotes "" and "". The quotes are not part of the string, but serve only to delimit it. The sequence "\t" and "\n" in the string is MATX notation for the tab and the newline character, respectively. Strings can be concatenated into larger strings with "+" operators.

12.1 Element and Elements

`S(1)`	1st element of a string S
`S(2:4)`	From 2nd to 4th elements of a string S

12.2 Setting Elements

`S(1) = "A"`	Set "A" to 1st element of S
`S(2:4) = "AB"`	Set "AB" between 2nd and 4th element of S

12.3 Comparison of strings

We can compare two strings by using relational operators `==, !=`, and the result is an integer 1 (true) or 0 (false).

```
string1 == string2  string1 is equal to string2
string1 != string2  string1 is not equal to string2
```

12.4 Evaluation of strings (matx)

Evaluation of a string which consists of any MATX codes can be done by using the function `eval()`. The return value is consistent with the evaluation result, i.e, the class of the return value depends on the string which should be evaluated. If the statement of MATX is evaluated instead of an expression, an integer 0 is always returned. For example, in the following statements

```
Real a, b, c;
Matrix A, B, C;

a = 1.0; b = 2.0;
A = [[1 2][3 4]]; B = [[5 6][7 8]];

c = eval("a + b");
C = eval("A + B");
```

the expression `eval("a + b")` calculates sum of real numbers `a` and `b`, whereas the expression `eval("A + B")` calculates the sum of matrices `A` and `B`.

It is very usefull for debuging the program to use the function `eval()` in a function as follows.

```
Func void afo()
{
  String cmd;

  ...................

  for (;;) {
    read cmd;
    if (cmd == "q") {
      break;
    }
    eval(cmd);
  }

  ...................
}
```

If we use the function `eval()` with the argument `"who"` or `"whos"`, we can see both local variables of the function and global variables.

```
Func void afo()
{
  ...................

  eval("who");
  eval("whos");

  ...................
}
```

12.5 String Functions

MATX provides the following string functions. "`printf()`" and "`fprintf()`" are general-purpose format conversion functions, which convert, formats, and output its arguments on the standard output and a specified file. The function "`sprintf()`" converts, formats, and creates strings from its arguments. Arguments may be integers, real numbers, and strings. The function "`fprintf()`" opens files in the append mode.


```
Integer printf(String format, ...);   Display (matx).
Integer fprintf(String file, String format, ...); To files.
String  sprintf(String format, ...);  Conversion to a string.
Integer length(String str);           Length of a string.
```

Chapter 13

Polynomial

Declaration:

```
Polynomial a, b, c;
```

Example:

```
2*s^2 + 3*s + 4,  (1,2)*s^2 + (3,4)*s + (5,6)
```

MATX handles polynomials of one variable with real and complex coefficients. Polynomials can be added, subtracted, multiplied, and divided as usual. At the moment, the coefficients of a polynomial are stored as a row vector containing the coefficients ordered by ascending powers. Polynomials are easily created and manipulated in MaTX.

13.1 How To Describe Polynomials

At least one polynomial variable must be defined first in order to describe polynomials. Polynomial variable is defined by using the symbol "`$`" and the character "`s`" is always used for the output polynomial variable. Integer coefficients are converted to real numbers. The class conversion function "`Polynomial()`" and "`Matrix()`" can be used to convert a row vector to a polynomial and visa versa. The coefficients of the polynomial are identical with the elements of the row vector in ascending order.


```
s  = $;                    /* Polynomial variable      */
P1 = 2*s^2 + 3*s + 4;      /* 2-degree real polynomial */
P2 = Polynomial([4 3 2]); /* P2 = 2*s^2 + 3*s + 4     */
CP1 = (1,2)*s + (3,4);     /* 1-deg complex polynomial */
```

13.2 Complex Polynomials

There are two ways to enter complex polynomials. The first is to describe polynomial with complex coefficients, and the second is to use two real polynomials separated by a "`,`" inside parentheses "`(`" and "`)`". Real or imaginary part can be omitted by specifying "`*`" or "`:`" as the other part.

```
CP1 = (P1,P2); /* CP1 = P1 + P2*i; */
CP1 = (P1,*);  /* CP1 = P1 + 0*i;  */
CP1 = (P1,:);  /* CP1 = P1 + 0*i;  */
CP1 = (*,P1);  /* CP1 = 0  + P1*i; */
CP1 = (:,P1);  /* CP1 = 0  + P1*i; */
```

, where `P1` and `P2` are real polynomials.
Real-part and imaginary-part of a complex polynomial can be extracted using the functions "`Re()`" and "`Im()`", respectively.

```
P1 = Re((P1, P2));  /* Real part      */
P2 = Im((P1, P2));  /* Imaginary part */
```

13.3 Coefficients

Individual polynomial coefficients can be specified with the index inside parentheses, "`(`" and "`)`". Real numbers and complex numbers can be assigned to coefficients of a complex polynomial. On the other hand, only

real numbers can be assigned to coefficients of a real polynomial. The index must be lower than the degree of the polynomial.

```
r1 = P1(1);   /* 1st-order coefficient of P1        */
c1 = CP1(2)   /* 2nd-order coefficient of CP1       */
P1(0) = r1;   /* Assign r1 to constant term         */
CP1(3) = c1;  /* Assign c1 to 3rd-order coefficient */
```

13.4 Evaluation of Polynomials

Evaluation of polynomials at a specified value can be done by using the function "`eval()`".

```
s  = $;
P1 = 3*s^2 + 4*s + 5;
r1 = eval(P1, 2.0);   /* Evaluation at 2.0          */
P2 = eval(P1, s + 1); /* Evaluation at "s + 1"      */
A2 = eval(P1, [1:3]); /* Evaluation at  [1, 2, 3]   */
M2 = eval(p1, M);     /* 3*M^2 + 4*M + 5*I(M)       */
```

Evaluation of a polynomial may involve a class conversion according to the following conversion rules.

```
Real number        = eval(Real-poly, Real number)
Complex number     = eval(Real-poly, Complex number)
Complex number     = eval(Comp-poly, Real number)
Complex number     = eval(Comp-poly, Complex number)
Real polynomial    = eval(Real-poly, Real-poly)
Complex polynomial = eval(Comp-poly, Real-poly)
Complex polynomial = eval(Real-poly, Comp-poly)
Real matrix        = eval(Real-poly, Real matrix)
Complex matrix     = eval(Comp-poly, Real matrix)
Complex matrix     = eval(Real-poly, Complex matrix)
Complex matrix     = eval(Comp-poly, Complex matrix)
Real array         = eval(Real-poly, Real array)
```



```
Complex array      = eval(Comp-poly, Real array)
Complex array      = eval(Real-poly, Complex array)
Complex array      = eval(Comp-poly, Complex array)
```

13.5 Derivative and Indefinite Integral

Derivatives and indefinite integrals of polynomials can be obtained by using the functions "`derivative()`" and "`integral()`", respectively. The order of the derivative and the indefinite integral can be specified as the argument of the functions. The function used without 2nd argument produce 1st derivative or 1st indefinite integral

```
s  = $;
P1 = 3*s^2 + 4*s + 5;
P2 = integral(P1);     /* 1st order indefinite integral */
P2 = integral(P1,2);   /* 2nd order indefinite integral */
P3 = derivative(P2);   /* 1st order derivative          */
P3 = derivative(P2,2); /* 2nd order derivative          */
```

13.6 Shift of Coefficients

The coefficients of the polynomials can be shifted toward the lower and higher order by using the functions "`lower()`" and "`higher()`", respectively. The number of shift can be specified as the argument of the functions. The function used without 2nd argument shift the coefficients once.

```
s  = $;
P1 = 3*s^2 + 4*s + 5;
P2 = higher(P1);    /* 1 shift  for the higher order */
P2 = higher(P1,2);  /* 2 shifts for the higher order */
P3 = lower(P2);     /* 1 shift  for the lower  order */
P3 = lower(P2,2);   /* 2 shifts for the lower  order */
```

13.7 Comparison of polynomials

We can compare two polynomials by using relational operators `==, !=`, and the result is an integer 1 (true) or 0 (false).

```
poly1 == poly2  poly1 is equal to poly2
poly1 != poly2  poly1 is not equal to poly2
```

13.8 Polynomial Functions

MATX provides the following polynomial functions.

```
Matrix  roots(Polynomial P1);     Root of polynomial
Integer degree(Polynomial P1);    Degree of polynomial
Integer isreal(Polynomial P1);    Classification of poly
                                  (True = 1, False = 0)
Integer iscomplex(Polynomial P1); Classification of poly
                                  (True = 1, False = 0)
```

Chapter 14

Rational Function

Declaration:

```
Rational a, b, c;
```

Example:

```
(s + 2)/(2*s + 3),  (1,2)/((3,4)*s + (5,6))
```

MATX handles rational-functions of one variable with real and complex coefficients. Rational-functions can be added, subtracted, multiplied, and divided as usual. At the moment, a rational-function is stored as a pair of polynomials, i.e., a numerator polynomial and a denominator polynomial. Rational-functions are easily created and manipulated in MaTX.

14.1 How To Describe Rational Functions

A rational-function is described as a fraction of a pair of polynomials, and the character "s" is always used for the output rational-function variable.

```
s  = $;
R1 = (s + 2)/(2*s + 3);
                    /* Real rational function    */
CR1 = (1,2)/((3,4)*s + (5,6));
                    /* Complex rational function */
```


14.2 Complex Rational Functions

There are two ways to enter complex rational functions. The first is to describe rational functions with complex coefficients, and the second is to use two real rational functions separated by a "," inside parentheses "(" and ")". Real or imaginary part can be omitted by specifying "*" or ":" as the other part.

```
CR1 = (R1,R2); /* CR1 = R1 + R2*i; (R1, R2:real rat) */
CR1 = (R1,*);  /* CR1 = R1 + 0*i;  (R1: real rat)    */
CR1 = (R1,:);  /* CR1 = R1 + 0*i;  (R1: real rat)    */
CR1 = (*,R1);  /* CR1 = 0  + R1*i; (R1: real rat)    */
CR1 = (:,R1);  /* CR1 = 0  + R1*i; (R1: real rat)    */
```

Real-part and imaginary-part of a complex rational-function can be extracted using the functions "Re()" and "Im()", respectively.

```
R1 = Re((R1, R2));  /* Real part      */
R2 = Im((R1, R2));  /* Imaginary part */
```

where

$$\mathtt{Re}\left(\frac{\mathtt{a}+\mathtt{b}i}{\mathtt{c}+\mathtt{d}i}\right)=\frac{\mathtt{ac}+\mathtt{bd}}{\mathtt{c}^2+\mathtt{d}^2},\quad \mathtt{Im}\left(\frac{\mathtt{a}+\mathtt{b}i}{\mathtt{c}+\mathtt{d}i}\right)=\frac{\mathtt{bc}-\mathtt{ad}}{\mathtt{c}^2+\mathtt{d}^2}$$

and, a, b, c, d are real polynomials.

$$\mathtt{Re}\left(\frac{\mathtt{a}+\mathtt{b}i}{\mathtt{c}+\mathtt{d}i}\right)=\frac{(\mathtt{a}+\mathtt{b}i)(\mathtt{c}-\mathtt{d}i)}{(\mathtt{c}+\mathtt{d}i)(\mathtt{c}-\mathtt{d}i)}=\frac{\mathtt{ac}+\mathtt{bd}}{\mathtt{c}^2+\mathtt{d}^2}+\frac{\mathtt{bc}-\mathtt{ad}}{\mathtt{c}^2+\mathtt{d}^2}i$$

14.3 Numerator and Denominator

Numerator and denominator of a rational-function can be extracted by using the functions, "Nu()" and "De()", respectively.

```
P1 = Nu(P1 / P2);  /* Numerator   */
P2 = De(P1 / P2);  /* Denominator */
```

14.4 Operations of Rational Functions

Operation of rational-functions are done according to the following rules, and no cancellation is applied to the operation. Please pay attention to the increase of degree of rational-functions.

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}$$

$$\frac{a}{b}*\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}/\frac{c}{d}=\frac{a\cdot d}{b\cdot c}$$

14.5 Evaluation

Evaluation of rational-functions at a specified value can be done by using the function "`eval()`".

```
s  = $;
R1 = (s + 2)/(2*s + 3);
r1 = eval(R1, 2.0);         /* Eval at 2.0             */
R2 = eval(R1, s + 1);       /* Eval at s+1             */
R2 = eval(R1, (s+1)/(s+2)); /* Eval at (s+1)/(s+2)     */
A2 = eval(R1, [1:4]);       /* Eval at [1,2,3,4]       */
M2 = eval(R1, M);           /* (M+2*I(M))/(2*M+3*I(M)) */
```

Evaluation of a rational-function may involve a class conversion according to the following conversion rules.

```
Real number      = eval(Real-rat-func, Real number)
Complex number   = eval(Real-rat-func, Complex number)
```



```
Complex number   = eval(Comp-rat-func, Real number)
Complex number   = eval(Comp-rat-func, Complex number)
Real rat-func    = eval(Real-rat-func, Real-poly)
Real rat-func    = eval(Real-rat-func, Real-rat-func)
Complex rat-func = eval(Real-rat-func, Comp-poly)
Complex rat-func = eval(Comp-rat-func, Real-poly)
Complex rat-func = eval(Real-rat-func, Comp-rat-func)
Complex rat-func = eval(Comp-rat-func, Real-rat-func)
Complex rat-func = eval(Comp-rat-func, Comp-rat-func)
Real matrix      = eval(Real-rat-func, Real matrix)
Complex matrix   = eval(Comp-rat-func, Real matrix)
Complex matrix   = eval(Real-rat-func, Complex matrix)
Complex matrix   = eval(Comp-rat-func, Complex matrix)
Real array       = eval(Real-rat-func, Real array)
Complex array    = eval(Comp-rat-func, Real array)
Complex array    = eval(Real-rat-func, Complex array)
Complex array    = eval(Comp-rat-func, Complex array)
```

14.6 Derivative

The function "`derivative()`" calculates the derivative of rational functions. The order of derivative can be specified by the arguments. The function with only 1 argument produces 1st order derivative. Indefinite integral of rational functions is not implemented for the moment.

```
s  = $;
R1 = (s + 2)/(2*s + 3);
R2 = derivative(R1);       /* 1st order derivative */
R2 = derivative(R1,2);     /* 2nd order derivative */
```

14.7 Shift of Coefficients

The coefficients of the rational-functions can be shifted toward the lower and higher order by using the functions "`lower()`" and "`higher()`", respectively. The number of shift can be specified as the argument of the

functions. The function used without 2nd argument shift the coefficients once.

```
s  = $;
R1 = (s + 2)/(2*s + 3);
R2 = higher(R1);    /* 1 shift  for the higher order */
R2 = higher(R1,2);  /* 2 shifts for the higher order */
R3 = lower(R2);     /* 1 shift  for the lower order  */
R3 = lower(R2,2);   /* 2 shifts for the lower order  */
```

14.8 Comparison of rational functions

We can compare two rational functions by using relational operators `==`, `!=`, and the result is an integer 1 (true) or 0 (false).

```
rat1 == rat2  rat1 is equal to rat2
rat1 != rat2  rat1 is not equal to rat2
```

14.9 Rational-Function Functions

MATX provides the following rational-function functions.

```
Matrix  zeros(Rational R1);     Zero of rational-functions
Matrix  poles(Rational R1);     Pole of rational-functions
Integer isreal(Rational R1);    Classification of rat-func
                                (True = 1, False = 0)
Integer iscomplex(Rational R1); Classification of rat-func
                                (True = 1, False = 0)
```

Chapter 15

Matrix, Array, and Index

Declaration:

```
Matrix A, B, C;
Array D, E, F;
Index II, JJ;
```

Example:

```
[[1 2][3 4]], [(1,2), (3,4)]
Array([[1 2][3 4]]), Array([(1,2), (3,4)])
Index([1 2 3 4]), Index([10 8 6 4 2])
```

MATX distinguishes Array class objects from Matrix class objects. Array class objects have the same data structure as that of Matrix class objects, but arithmetic operations of arrays are element-by-element in stead of the usual algebraic matrix operations. For array operations of matrices, Matrix class has array operators denoted by the symbols, `.*`, `./`, `.\`, `.^`. The functions "`Matrix()`" is used to convert Matrix class objects to Array class object and the function "`Array()`" vice versa. The subscripting abilities of MATX allow manipulation of rows, columns, individual elements, and submatrices of matrices.

Index, which consists of integers, is usefull for the operation, such as assignment and retrieving of submatrices from a matrix. An index is described by using the class conversion function "`Index()`".

15.1 How To Describe Matrices

The easiest method of entering small matrices is to use explicit lists. The explicit list of elements separated by blanks, tabs, commas "`,`", or carriage returns, is surrounded by square brackets "`[`" and "`]`". The element which contains the unary − operator must be separated by commas or surrounded by parentheses, "`(`" and "`)`". "`A = []`" denotes a `0 x 0` matrix. Brackets are also used for indicating the begins and ends of the rows. Arbitrary number of spaces and carriage returns may be inserted in matrices. This enables us to describe the program in a natural fashion. Big matrices can be constructed by using small matrices as elements.

15.1.1 Conversion Rule of Elements

If there are distinct class elements in a matrix, class conversion is done according to the following rules.

1. Integers are converted to real numbers.
2. If there are complex numbers or complex matrices, all elements are converted to complex numbers.
3. If there are polynomials or polynomial matrices, all elements are converted to polynomials.
4. If there are rational-functions or rational-function matrices, all elements are converted to rational-functions.

EXCEPTION If all elements are strings, they are concatenated to a string. (This exception exists only for the compatibility with MATLAB. Users should not use this expression.)

If there are some expressions in the element, those expression are evaluated at first and the class conversion follows. Please be carefull not to make the following mistake:

```
M1 = [1/2, 3/4, 4/5, 5/6];
```

The result is totally different from the one expected, and becomes a 1×4 zero matrix. In order to obtain a desirable resut one has to describe as

```
M1 = [1./2., 3./4., 4./5., 5./6.];
```

15.1.2 Row Vector

Row vectors are described as the explicit list of elements separated by blanks, tabs, commas ",", or carriage returns, and which is surrounded by square brackets "[" and "]".

```
M1 = [];            /* 0 x 0 matrix                    */
M1 = [1 2 3];       /* Separated by blanks             */
M1 = [1 2 , -3];    /* - operator separated by a comma */
M1 = [1 2 (-3)];    /* - operator inside parentheses   */
CM1 = [(1,2), (3,4)]; /* Complex row vector            */
```

15.1.3 Column Vector

A single quote " ' " is used to transpose a row vector.

```
M1 = [1 2 3]';       /* Separated by blanks             */
M1 = [1 2 , -3]';    /* - operator separated by a comma */
M1 = [1 2 (-3) ]';   /* - operator inside parentheses   */
CM1 = [(1,2), (3,4)]'; /* Complex column vector         */
```

15.1.4 Matrices

The way of creating matrices is with some row vectors.


```
M1 = [[1 2 3][4,-5 6][7 (-8), -9]];

M1 = [[1  2     3]    CM1 = [[(1,2) (3,4)]
      [4 ,-5    6]           [(5,6) (7,8)]];
      [7 (-8), -9]];
```

15.2 Miscellaneous Matrices

MATX provides a low of ways to enter various kinds of matrices..

15.2.1 Zero Matrices

The piece matrix is used to specify the unit size of big zero matrices.

```
M1 = Z(3);        3x3 real matrix.
M1 = Z(3, c);     3x3 complex matrix,   c = Complex Number.
M1 = Z(3, P);     3x3 real-poly matrix,      P = Real-Poly.
M1 = Z(3, CP);    3x3 comp-poly matrix,     CP = Comp-Poly.
M1 = Z(3, R);     3x3 real-rat-func matrix,   R = Real-Rat.
M1 = Z(3, CR);    3x3 comp-rat-func matrix,  CR = Comp-Rat.
M1 = Z(3, 4);     3x4 real matrix.
M1 = Z(3, 4, c);  3x4 complex matrix,   c = Complex Number.
M1 = Z(3, 4, P);  3x4 real-poly matrix,      P = Real-Poly.
M1 = Z(3, 4, CP); 3x4 comp-poly matrix,     CP = Comp-Poly.
M1 = Z(3, 4, R);  3x4 real-rat-func matrix,   R = Real-Rat.
M1 = Z(3, 4, CR); 3x4 comp-rat-func matrix, CR = Comp-Rat.
M2 = Z(M1);       Same size and same class as matrix M1.
M2 = Z(3, M1);    Same class and three times
                  as large as matrix M1.
M2 = Z(2, 3, M1); Same class and two times by columns and
                  three times by rows as matrix M1.
```

15.2.2 Identity Matrices

In case of nonsquare matrix, all diagonal elements are 1 and others are 0. In case of polynomial matrix, all diagonal elements are polynomials whose constant term is 1 and other coefficients are 0. In case of rational function matrix, all diagonal elements are rational functions whose constant term is 1 and other coefficients are 0. The piece matrix is used to specify the unit size of big identify matrices.

```
M1 = I(3);          3x3 real matrix.
M1 = I(3, c);       3x3 complex matrix,  c = Complex Number.
M1 = I(3, P);       3x3 real-poly matrix,     P = Real-Poly.
M1 = I(3, CP);      3x3 comp-poly matrix,    CP = Comp-Poly.
M1 = I(3, R);       3x3 real-rat-func matrix,  R = Real-Rat.
M1 = I(3, CR);      3x3 comp-rat-func matrix, CR = Comp-Rat.
M1 = I(3, 4);       3x4 real matrix,
M1 = I(3, 4, c);    3x4 complex matrix,  c = Complex Number.
M1 = I(3, 4, P);    3x4 real-poly matrix,     P = Real-Poly.
M1 = I(3, 4, CP);   3x4 comp-poly matrix,    CP = Comp-Poly.
M1 = I(3, 4, R);    3x4 real-rat-func matrix,  R = Real-Rat.
M1 = I(3, 4, CR);   3x4 comp-rat-func matrix, CR = Comp-Rat.
M2 = I(M1);         Same size and same class as matrix M1.
M2 = I(3, M1);      Same class and three times
                    as large as matrix M1.
M2 = I(2, 3, M1);   Same class and two times by columns and
                    three times by rows as matrix M1.
```

15.2.3 Matrix Filled with "1"

In case of polynomial matrix, all elements are polynomials whose constant term is 1 and other coefficients are 0. In case of rational function matrix, all elements are rational functions whose constant term is 1 and other coefficients are 0. The piece matrix is used to specify the unit size of big matrices.

```
M1 = ONE(3);          3x3 real matrix.
M1 = ONE(3, c);       3x3 complex matrix,  c = Complex Number
```



```
M1 = ONE(3, P);       3x3 real-poly matrix,     P = Real-Poly
M1 = ONE(3, CP);      3x3 comp-poly matrix,    CP = Comp-Poly
M1 = ONE(3, R);       3x3 real-rat-func matrix,  R = Real-Rat
M1 = ONE(3, CR);      3x3 comp-rat-func matrix, CR = Comp-Rat
M1 = ONE(3, 4);       3x4 real matrix.
M1 = ONE(3, 4, c);    3x4 complex matrix,  c = Complex Number
M1 = ONE(3, 4, P);    3x4 real-poly matrix,     P = Real-Poly
M1 = ONE(3, 4, CP);   3x4 comp-poly matrix,    CP = Comp-Poly
M1 = ONE(3, 4, R);    3x4 real-rat-func matrix,  R = Real-Rar
M1 = ONE(3, 4, CR);   3x4 comp-rat-func matrix, CR = Comp-Rat
M2 = ONE(M1);         Same size and same class as matrix M1
M2 = ONE(3, M1);      Same class and three times
                      as large as matrix M1
M2 = ONE(2, 3, M1);   Same class and two times by columns and
                      three times by rows as matrix M1
```

15.2.4 Arithmetic Sequence Matrices

Arithmetic sequence can be created by using the special character, the colon ":", in side square brackets, "[" and "]". The vectors created by the colon notation are not matrices but arrays.

```
/* Row array containing the numbers from a1 to a3 by a2 */
A1 = [a1:a2:a3];

/* Row array containing the numbers from a1 to a2 by 1 */
A1 = [a1:a2];

A1 = [0.1:0.1:0.5];  /* M1 = [0.1 0.2 0.3 0.4 0.5]  */
A2 = [0.5:-0.1:0.1]; /* M2 = [0.5 0.4 0.3 0.2 0.1]  */
A3 = [4:12];         /* M3 = [4 5 6 7 8 9 10 11 12] */
```

15.2.5 Diagonal and Block-Diagonal Matrices

In defining a diagonal or block-diagonal matrix, all elements must be the same class. Scalar values, such as integer, real number, and polynomials,

should be converted to `1 x 1` matrix to define a block-diagonal matrix.

```
M1 = diag(1.0, 2.0, 3.0); /* Real diagonal matrix    */
CM1 = diag((1,2), (3,4)); /* Complex diagonal matrix */
M4 = diag(M1, M2, M3);    /* Block-diagonal matrix   */
M3 = diag(M1, M2, [1]);   /* Block-diagonal matrix   */
```

15.2.6 Complex Matrices

There are at least two convenient ways to enter complex matrices. One uses a pair of real matrices as the real-part and the imaginary-part of a matrix. The other uses complex numbers as elements of a matrix.

The real-part or imaginary-part can be omitted by using "*" or ":" instead.

```
CM3 = (M1,M2); /* CM3 = M1 + M2*i;     */
CM3 = (M1,*);  /* CM3 = M1 + Z(M1)*i;  */
CM3 = (M1,:);  /* CM3 = M1 + Z(M1)*i;  */
CM3 = (*,M1);  /* CM3 = Z(M1)*i + M1;  */
CM3 = (:,M1);  /* CM3 = Z(M1)*i + M1;  */
```

, where `M1` and `M2` are real matrices.

Real-part and imaginary part of a complex matrix can be extracted by using the functions "`Re()`" and "`Im()`", respectively.

```
M1 = Re((M1,M2));  /* Real part matrix      */
M2 = Im((M1,M2));  /* Imaginary part matrix */
```

15.2.7 Composed Matrix

Big matrices can be constructed by using little matrices as elements. It should be noted that the size of the individual element matrices are appropriate.


```
M1 = [[1 2][3 4]];  M2 = [5 6]';  M3 = [7 8];
M4 = [9];  M5 = [[1 0][0 1]];  M6 = [1];

M7 = [[ M1   , -M2*M6~*M2#]
      [-M5   , -M1#       ]];
```

15.3 Polynomial Matrix

15.3.1 How To Describe Polynomial Matrices

Essentially the manipulation of polynomial matrices is identical with that for manipulation for numerical matrices, i.e., real matrices and complex matrices. There are at least two convenient ways to enter polynomial matrices. One uses polynomials as elements of a matrix. The other uses matrices as coefficients of a polynomial.

```
s   = $;

PM1 = [[s + 1  s + 2]    PM2 = [[s + 5  s + 6]
       [s + 3  s + 4]];         [s + 7  s + 8]];

PM1 = diag(s+1, s+2, s+3, s+4);
```

There are at least two convenient ways to enter complex polynomial matrices. One uses a pair of real polynomial matrices as the real-part and the imaginary-part of a matrix. The other uses complex polynomials as elements of a matrix.

The real-part or imaginary-part can be omitted by using "*" or ":" instead.

```
CPM1 = (PM1,PM2); /* CPM1 = PM1 + PM2 i,          */
                  /* PM1, PM2: real poly matrices */
CPM1 = (PM1,*);   /* CPM1 = PM1 + Z(PM1) i,       */
                  /* PM1: real polynomial matrix. */
CPM1 = (PM1,:);   /* CPM1 = PM1 + Z(PM1) i,       */
                  /* PM1: real polynomial matrix. */
CPM1 = (*,PM1);   /* CPM1 = Z(PM1) + PM1 i,       */
                  /* PM1: real polynomial matrix. */
CPM1 = (:,PM1);   /* CPM1 = Z(PM1) + PM1 i,       */
                  /* PM1: real polynomial matrix. */
```

Real-part and imaginary part of a complex polynomial matrix can be extracted by using the functions "`Re()`" and "`Im()`", respectively.

```
PM1 = Re((PM1, PM2));  /* Real part matrix      */
PM2 = Im((PM1, PM2));  /* Imaginary part matrix */
```

Create a complex polynomial matrix from two real polynomial matrices.

```
s   = $;

PM1 = [[s + 1  s + 2]   PM2 = [[s + 5  s + 6]
       [s + 3  s + 4]];        [s + 7  s + 8]];

CPM1 = (PM1, PM2);
CPM2 = [[(1,1)*s+(1,5) (1,1)*s+(2,6)]
        [(1,1)*s+(3,7) (1,1)*s+(4,8)]];
```

15.3.2 Evaluation of Polynomial Matrices

Evaluation of polynomial matrices at a specified value can be done by using the function "`eval()`". If the argument is an array, it is substituted to the variable of each element of a polynomial matrix. For example, substituting a matrix into the variable


```
s   = $;
RM1 = [[(s+1) (s+2)]
       [(s+3) (s+4)]];

A = [[1 2][3 4]];

B = eval(RM1, A);
```

make a matrix `B` as

$$\mathtt{B} = \begin{bmatrix} \mathtt{eval(s+1,\ A)} & \mathtt{eval(s+2,\ A)} \\ \mathtt{eval(s+3,\ A)} & \mathtt{eval(s+4,\ A)} \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 & 2 \\ 3 & 5 & 3 & 6 \\ 4 & 2 & 5 & 2 \\ 3 & 7 & 3 & 8 \end{bmatrix}$$

With an array,

```
s   = $;
RM1 = [[(s+1) (s+2)]
       [(s+3) (s+4)]];

A = [[1 2][3 4]];

B = eval(RM1, Array(A));
```

make an array `B` as

$$\mathtt{B} = \begin{bmatrix} \mathtt{eval(RM1,\ 1)} & \mathtt{eval(RM1,\ 2)} \\ \mathtt{eval(RM1,\ 3)} & \mathtt{eval(RM1,\ 4)} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 & 4 \\ 4 & 5 & 5 & 6 \\ 4 & 5 & 5 & 6 \\ 6 & 7 & 7 & 8 \end{bmatrix}$$

The following is a class conversion rule in the evaluation of polynomial matrices.

```
Real Matrix      = eval(Real-Poly Matrix, Real Number)
Complex Matrix   = eval(Real-Poly Matrix, Complex Number)
Complex Matrix   = eval(Comp-Poly Matrix, Real Number)
Complex Matrix   = eval(Comp-Poly Matrix, Complex Number)
Real-Poly Matrix = eval(Real-Poly Matrix, Real-Poly)
Comp-Poly Matrix = eval(Comp-Poly Matrix, Real-Poly)
Comp-Poly Matrix = eval(Real-Poly Matrix, Comp-Poly)
Real-Rat Matrix  = eval(Real-Poly Matrix, Real-Rat)
Comp-Rat Matrix  = eval(Comp-Poly Matrix, Real-Rat)
Comp-Rat Matrix  = eval(Real-Poly Matrix, Comp-Rat)
Comp-Rat Matrix  = eval(Comp-Poly Matrix, Comp-Rat)
```

```
Real Matrix      = eval(Real-Poly Matrix, Real Mat)
Complex Matrix   = eval(Real-Poly Matrix, Complex Mat)
Complex Matrix   = eval(Comp-Poly Matrix, Real Mat)
Complex Matrix   = eval(Comp-Poly Matrix, Complex Mat)
Real-Poly Matrix = eval(Real-Poly Matrix, Real-Poly Mat)
Comp-Poly Matrix = eval(Comp-Poly Matrix, Real-Poly Mat)
Comp-Poly Matrix = eval(Real-Poly Matrix, Comp-Poly Mat)
Real-Rat Matrix  = eval(Real-Poly Matrix, Real-Rat Mat)
Comp-Rat Matrix  = eval(Comp-Poly Matrix, Real-Rat Mat)
Comp-Rat Matrix  = eval(Real-Poly Matrix, Comp-Rat Mat)
Comp-Rat Matrix  = eval(Comp-Poly Matrix, Comp-Rat Mat)
```

```
Real Array      = eval(Real-Poly Matrix, Real Arr)
Complex Array   = eval(Real-Poly Matrix, Complex Arr)
Complex Array   = eval(Comp-Poly Matrix, Real Arr)
Complex Array   = eval(Comp-Poly Matrix, Complex Arr)
Real-Poly Array = eval(Real-Poly Matrix, Real-Poly Arr)
Comp-Poly Array = eval(Comp-Poly Matrix, Real-Poly Arr)
Comp-Poly Array = eval(Real-Poly Matrix, Comp-Poly Arr)
Real-Rat Array  = eval(Real-Poly Matrix, Real-Rat Arr)
Comp-Rat Array  = eval(Comp-Poly Matrix, Real-Rat Arr)
Comp-Rat Array  = eval(Real-Poly Matrix, Comp-Rat Arr)
Comp-Rat Array  = eval(Comp-Poly Matrix, Comp-Rat Arr)
```


15.3.3 Derivative and Indefinite Integral

Derivatives and indefinite integrals of polynomial matrices can be obtained by using the functions "`derivative()`" and "`integral()`", respectively. The order of the derivative and the indefinite integral can be specified as the argument of the functions.

```
s   = $;
PM1 = [[s + 1  s + 2]    PM2 = [[s + 5  s + 6]
       [s + 3  s + 4]];         [s + 7  s + 8]];

CPM1 = (PM1, PM2);
CPM2 = integral(CPM1);     /* 1st order indefinite */
CPM3 = integral(CPM1,2);   /* 2nd order indefinite */
CMP4 = derivative(CMP2);   /* 1st order derivative */
CMP5 = derivative(CMP3,2); /* 2nd order derivative */
```

15.3.4 Shift of Coefficients

The coefficients of the polynomial matrices can be shifted toward the lower and higher order by using the functions "`lower()`" and "`higher()`", respectively. The number of shift can be specified as the argument of the functions.

```
s   = $;
PM1 = [[s + 1  s + 2]    PM2 = [[s + 5  s + 6]
       [s + 3  s + 4]];         [s + 7  s + 8]];
CPM1 = (PM1, PM2);

CPM2 = higher(CPM1);   /* One shift to higher order  */
CPM3 = higher(CPM1,2); /* Two shifts to higher order */
CPM4 = lower(CPM2);    /* One shift to lower order   */
CPM5 = lower(CPM3,2);  /* Two shift to lower order   */
```

15.4 Rational Function Matrix

15.4.1 How To Describe Rational Function Matrices

Essentially the manipulation of rational function matrices is identical with that for manipulation for numerical matrices, i.e., real matrices and complex matrices. There are at least two convenient ways to enter rational function matrices. One uses rational functions as elements of a matrix. The other uses two polynomial matrices and an array operation.

```
s   = $;

RM1 = [[(s+1)/(s+2)  (s+3)/(s+4)]
       [(s+5)/(s+6)  (s+7)/(s+8)]];

RM2 = diag((s+1)/(s+2), (s+3)/(s+4), (s+5)/(s+6));

A = [[1 3][5 7]];  B = [[2 4][6 8]];
C = (ONE(2)*x + A) ./ (ONE(2)*x + B);
```

There are at least two convenient ways to enter complex rational function matrices. One uses a pair of real rational function matrices as the real-part and the imaginary-part of a matrix. The other uses complex rational functions as elements of a matrix.

The real-part or imaginary-part can be omitted by using "*" or ":" instead.


```
CRM1 = (RM1,RM2); /* CRM1 = RM1 + RM2 i,          */
                  /* RM1, RM2: real-rat matrices. */
CRM1 = (RM1,*);   /* CRM1 = RM1 + Z(RM1) i,       */
                  /* RM1: real-rat matrix.        */
CRM1 = (RM1,:);   /* CRM1 = RM1 + Z(RM1) i,       */
                  /* RM1: real-rat matrix.        */
CRM1 = (*,RM1);   /* CRM1 = Z(RM1) + RM1 i,       */
                  /* RM1: real-rat matrix.        */
CRM1 = (:,RM1);   /* CRM1 = Z(RM1) + RM1 i,       */
                  /* RM1: real-rat matrix.        */
```

Real-part and imaginary part of a complex rational matrix can be extracted by using the functions "`Re()`" and "`Im()`", respectively.

```
RM1 = Re((RM1, RM2));  /* Real part matrix      */
RM2 = Im((RM1, RM2));  /* Imaginary part matrix */
```

Create a complex rational function matrix from two real rational function matrices.

```
s   = $;
RM1 = [[(s+1)/(s+2)  (s+3)/(s+4)]
       [(s+5)/(s+6)  (s+7)/(s+8)]];

RM2 = [[(s+3)/(s+2)  (s+5)/(s+4)]
       [(s+7)/(s+6)  (s+1)/(s+8)];

CRM1 = (RM1, RM2);

CRM2 = [[((1,1)*s+(1,3))/(s+2) ((1,1)*s+(3,5))/(s+4)]
        [((1,1)*s+(5,7))/(s+6) ((1,1)*s+(7,1))/(s+8)]];
```

15.4.2 Evaluation of Rational Matrices

Evaluation of rational function matrices at a specified value can be done by using the function "`eval()`". If the argument is an array, it is substituted to the variable of each element of a rational function matrix. For example, substituting a matrix into the variable,

```
s   = $;
RM1 = [[(s+1)/(s+2)  (s+3)/(s+4)]
       [(s+5)/(s+6)  (s+7)/(s+8)]];

A = [[1 2][3 4]];

B = eval(RM1, A);
```

make a matrix `B` as

$$\mathtt{B} = \begin{bmatrix} \mathtt{eval((s+1)/(s+2),\ A)} & \mathtt{eval((s+3)/(s+4),\ A)} \\ \mathtt{eval((s+5)/(s+6),\ A)} & \mathtt{eval((s+7)/(s+8),\ A)} \end{bmatrix}$$

$$= \begin{bmatrix} 0.66667 & 0.75000 & 0.80000 & 0.83333 \\ 0.80000 & 0.83333 & 0.85714 & 0.87500 \\ 0.85714 & 0.87500 & 0.88889 & 0.90000 \\ 0.88889 & 0.90000 & 0.90909 & 0.91667 \end{bmatrix}$$

With an array,

```
s   = $;
RM1 = [[(s+1)/(s+2)  (s+3)/(s+4)]
       [(s+5)/(s+6)  (s+7)/(s+8)]];

A = [[1 2][3 4]];

B = eval(RM1, Array(A));
```

make an array `B` as

$$\mathtt{B} = \begin{bmatrix} \mathtt{eval(RM1,\ 1)} & \mathtt{eval(RM1,\ 2)} \\ \mathtt{eval(RM1,\ 3)} & \mathtt{eval(RM1,\ 4)} \end{bmatrix}$$

$$= \begin{bmatrix} 0.66667 & 0.75000 & 0.80000 & 0.83333 \\ 0.80000 & 0.83333 & 0.85714 & 0.87500 \\ 0.85714 & 0.87500 & 0.88889 & 0.90000 \\ 0.88889 & 0.90000 & 0.90909 & 0.91667 \end{bmatrix}$$

The following is a class conversion rule in the evaluation of rational function matrices.

```
Real Matrix     = eval(Real-Rat Matrix, Real Number)
Complex Matrix  = eval(Real-Rat Matrix, Complex Number)
Complex Matrix  = eval(Comp-Rat Matrix, Real Number)
Complex Matrix  = eval(Comp-Rat Matrix, Complex Number)
Real-Rat Matrix = eval(Real-Rat Matrix, Real-Poly)
Real-Rat Matrix = eval(Real-Rat Matrix, Real-Rat)
Comp-Rat Matrix = eval(Real-Rat Matrix, Comp-Poly)
Comp-Rat Matrix = eval(Comp-Rat Matrix, Real-Poly)
Comp-Rat Matrix = eval(Real-Rat Matrix, Comp-Rat)
Comp-Rat Matrix = eval(Comp-Rat Matrix, Real-Rat)
Comp-Rat Matrix = eval(Comp-Rat Matrix, Comp-Rat)

Real Matrix     = eval(Real-Rat Matrix, Real Mat)
Complex Matrix  = eval(Real-Rat Matrix, Complex Mat)
Complex Matrix  = eval(Comp-Rat Matrix, Real Mat)
Complex Matrix  = eval(Comp-Rat Matrix, Complex Mat)
Real-Rat Matrix = eval(Real-Rat Matrix, Real-Poly Mat)
Real-Rat Matrix = eval(Real-Rat Matrix, Real-Rat Mat)
Comp-Rat Matrix = eval(Real-Rat Matrix, Comp-Poly Mat)
Comp-Rat Matrix = eval(Comp-Rat Matrix, Real-Poly Mat)
Comp-Rat Matrix = eval(Real-Rat Matrix, Comp-Rat Mat)
Comp-Rat Matrix = eval(Comp-Rat Matrix, Real-Rat Mat)
Comp-Rat Matrix = eval(Comp-Rat Matrix, Comp-Rat Mat)

Real Array     = eval(Real-Rat Matrix, Real Arr)
Complex Array  = eval(Real-Rat Matrix, Complex Arr)
Complex Array  = eval(Comp-Rat Matrix, Real Arr)
Complex Array  = eval(Comp-Rat Matrix, Complex Arr)
Real-Rat Array = eval(Real-Rat Matrix, Real-Poly Arr)
Real-Rat Array = eval(Real-Rat Matrix, Real-Rat Arr)
Comp-Rat Array = eval(Real-Rat Matrix, Comp-Poly Arr)
```

```
Comp-Rat Array = eval(Comp-Rat Matrix, Real-Poly Arr)
Comp-Rat Array = eval(Real-Rat Matrix, Comp-Rat Arr)
Comp-Rat Array = eval(Comp-Rat Matrix, Real-Rat Arr)
Comp-Rat Array = eval(Comp-Rat Matrix, Comp-Rat Arr)
```

15.4.3 Derivative

Derivatives of rational function matrices can be obtained by using the functions "`derivative()`". The order of the derivative can be specified as the argument of the functions. At the moment, the indefinite integral of rational function matrices is not implemented.

```
s   = $;
RM1 = [[(s+1)/(s+2)  (s+3)/(s+4)]
       [(s+5)/(s+6)  (s+7)/(s+8)]];

RM2 = [[(s+3)/(s+2)  (s+5)/(s+4)]
        [(s+7)/(s+6)  (s+1)/(s+8)]];

CRM1 = (RM1, RM2);

CRM2 = derivative(CRM1);   /* 1st order derivative */
CRM3 = derivative(CRM1,2); /* 2nd order derivative */
```

15.4.4 Shift of Coefficients

The coefficients of the rational matrices can be shifted toward the lower and higher order by using the functions "`lower()`" and "`higher()`", respectively. The number of shift can be specified as the argument of the functions.


```
s   = $;
RM1 = [[(s+1)/(s+2)  (s+3)/(s+4)]
       [(s+5)/(s+6)  (s+7)/(s+8)]];

RM2 = [[(s+3)/(s+2)  (s+5)/(s+4)]
        [(s+7)/(s+6)  (s+1)/(s+8)]];

CRM1 = (RM1, RM2);

CRM2 = higher(CRM1);   /* One shift to higher order  */
CRM3 = higher(CRM1,2); /* Two shifts to higher order */
CRM4 = lower(CRM2);    /* One shift to lower order   */
CRM5 = lower(CRM3,2);  /* Two shifts to lower order  */
```

15.4.5 Numerator Array and Denominator Array

The function "`Nu()`" and "`De()`" extract the numerator part and the denominator part from a rational function arrays, respectively.

```
s = $;
A = [[1 2]     B = [[5 6]
     [3 4]];        [7 8]];

C = (ONE(2)*s + A) ./ (ONE(2)*s + B);

 /* C = [[(s+1)/(s+5)  (s+2)/(s+6)]   */
 /*      [(s+3)/(s+7)  (s+4)/(s+8)]]; */

N = Nu(Array(C));  /* N = [[s+1  s+2]   */
                   /*      [s+3  s+4]]; */

D = De(Array(C));  /* D = [[s+5  s+6]   */
                   /*      [s+7  s+8]]; */
```

15.5 Manipulation of Elements

The subscripting abilities in MATX allow manipulation of rows, columns, individual elements, and submatrices of matrices. Individual matrix elements can be specified with indices separated by a comma "," inside parentheses, "(" and ")". In case of the manipulation of the first row, the row number can be omitted. Any submatrices can be extracted from a big matrix specifying the subportions with the left-top and right-bottom locations, or the intervals of rows and columns with :.

Index enables us to make submatrices form arbitrary elements of a matrix. It is also possible to assign elements of a matrix to arbitrary position of another matrix by using Index.

The piece matrix enables us to treat a big matrix as an array of a small matrix. Index and piece-matrix can be used at the same time for the operation of an arbitrary array of matrix.

15.5.1 Elements and Submatrices

`M1(1,1)`	(1,1)-element
`M1(5)`	The 5th elements in the order: (1,1) → (1,Cols(M1)) → (2,1) → (2,Cols(M1)) → ⋯ → (Rows(M1),Cols(M1))
`M1(3:5)`	The vector of 3-5th elements
`M1(1,*)`	The first row vector
`M1(1,:)`	The first row vector
`M1(*,1)`	The first column vector
`M1(:,1)`	The first column vector
`M1(0,M2)`	The first submatrix according to the piece matrix M2
`M1(1,M2)`	The second submatrix according to the piece matrix M2
`M1(0,1,M2)`	The (1,2)-submatrix according to the piece matrix M2
`M1(1,2,M2)`	The (2,3)-submatrix according to the piece matrix M2

`M1(1:2,2:3)`	1st-2nd rows and 2nd-3rd columns
`M1(2:3,*)`	2nd-3rd rows
`M1(2:3,:)`	2nd-3rd rows
`M1(2:3,4)`	2nd-3rd rows and 4th column
`M1(*,2:3)`	2nd-3rd columns
`M1(:,2:3)`	2nd-3rd columns
`M1(4,2:3)`	4th row and 2nd-3rd columns

15.5.2 Setting Elements

Real numbers as well as complex numbers can be assigned to a complex matrix. However, only real numbers may be assigned to a real matrix.

`M1(1,1) = 1.0;`	Set 1.0 to (1,1)-element
`M1(2) = 1.0;`	Set 1.0 to (1,2)-element At the first row, the row number can be omitted
`CM1(1,1) = (1, 2);`	Set (1, 2) to (1,1)-element
`M1(0,0,M2) = M3;`	(1,1) is the beginning of right-hand side matrix according to the piece matrix M2
`M1(1,2,M2) = M3;`	(2,3) is the beginning of right-hand side matrix according to the piece matrix M2
`M1(1,M2) = M3;`	(1,2) is the beginning of right-hand side matrix according to the piece matrix M2
`M1(2,M2) = M3;`	(1,3) is the beginning of right-hand side matrix according to the piece matrix M2
`M1(3:5) = M3;`	Set martix M3 at 3rd-5th elements of matrix M1
`M1(1,*) = M3;`	Set martix M3 at 1st row of matrix M1
`M1(1,:) = M3;`	Set martix M3 at 1st row of matrix M1
`M1(*,2) = M3;`	Set martix M3 at 2nd column of matrix M1

`M1(:,2) = M3;`	Set martix M3 at 2nd column of matrix M1
`M1(1:2,1:2) = M3;`	Set matrix M3 at 1st-2nd row and 1st-2nd columns of matrix M1
`M1(1:,1:) = M3;`	(1,1) is the beginning of M3
`M1(1:,1) = M3;`	(1,1) is the beginning of M3
`M1(1,1:) = M3;`	(1,1) is the beginning of M3
`M1(1,2:3) = M3;`	Set matrix M3 at 1st row and 2nd-3rd columns of matrix M1
`M1(*,2:3) = M3;`	Set matrix M3 at 2nd-3rd columns of M1
`M1(:,2:3) = M3;`	Set matrix M3 at 2nd-3rd columns of M1
`M1(2:3,1) = M3;`	Set matrix M3 at 2nd-3rd rows and 1st column of matrix M1
`M1(2:3,*) = M3;`	Set matrix M3 at 2nd-3rd rows of M1
`M1(2:3,:) = M3;`	Set matrix M3 at 2nd-3rd rows of M1

15.5.3 Removing rows and columns

An efficient way of removing rows and columns of a matrix is to assign them to an empty matrix.

`M1(1,*) = [];`	Delete 1st row of M1
`M1(2:3,*) = [];`	Delete 2nd and 3rd rows of M11
`M1(*,1) = [];`	Delete 1st column of M1
`M1(*,2:3) = [];`	Delete 2nd and 3rd columns of M11
`M1(2:3) = [];`	Delete (1,2)-,(1,3)-element of M1
	At the first row, the row number can be omitted

15.5.4 Reshape of Matrices

The dimensions of matrices can be changed by using the function `reshape()` with the new dimensions of a matrix. The element ordering in the matrix is left to right and top to bottom which explains the ordering of the rearranged matrix in M1.


```
M1 = reshape([1 2 3 4], 2, 2); /* M1 = [[1 2][3 4]] */
```

15.5.5 Operation of elements by Index

Index, which consists of integers, is usefull for the operation, such as assignment and retrieving of submatrices from a matrix. An index is described by using the class conversion function "`Index()`". Double or triple integers separated by colons, which is used for setting and getting of elements from an matrix, have similar function as Index.

If `A` is a `n`-dimenstional vector and `v` is a `n`-dimensional index, `A(v)` is

```
[A(v(1)), A(v(2)), A(v(3)), ..., A(v(n))]
```

More sophisticated effects are obtained by using index referencing on both sides of an assignment statement. For instance

```
A(*, Index([3 5 10])) = B(*, 1:3)
```

replaces the thired, fifth, and tenth columns of `A` with the first three columns of `B` In general, if `v` and tt `w` are indices,

```
A(v, w)
```

is the matrix obtained by taking the elements of `A` with row subscripts from `v` and column subscripts from `w`. So,

```
A(*, n:-1:1)
```

reverses the columns of `A`.

Index can be used with a piece-matrix for the operation of an array of matrices For example, if `A` is an `n` $\times$ `n` matrix, `AA` is an `n` $\times$ `n`^{2} array of matrix, and `v` is an `n`-dimensional index, `AA(v, A)` is

```
[AA(v(1), A), AA(v(2), A), AA(v(3), A), ..., AA(v(n), A)]
```

More sophisticated effects are obtained by using index referencing on both sides of an assignment statement. For instance

```
AA(*, Index([3 5 10]), A) = BB(*, 1:3, A)
```

replaces the thired, fifth, and tenth columns of `AA` with the first three columns of `BB` In general, if `v` and tt w are indices,

```
AA(v, w, A)
```

is the sub-matrix obtained by taking the block-elements of `AA` with row subscripts from `v` and column subscripts from `w`. So,

```
AA(*, n:-1:1, A)
```

reverses the columns of `AA`.

`M1(IDX2)`	Submatrix with IDX2-elements of M1
`M1(1,IDX2)`	Submatrix with 1st-row, IDX2-col of M1
`M1(IDX1,IDX2)`	Submatrix with IDX1-row, IDX2-col of M1
`M1(IDX1,1)`	Submatrix with IDX1-row, 1st-col of M1
`M1(IDX1,1:2)`	Submatrix with IDX1-row, 1st-2nd-col of M1
`M1(1:2,IDX2)`	Submatrix with 1st-2nd-row, IDX2-col of M1
`MM(IDX2,M)`	Submatrix 1st-row, IDX2-col for a piece matrix M of MM
`MM(IDX1,IDX2,M)`	Submatrix IDX1-row, IDX2-col for a piece matrix M of MM
`MM(1,IDX2,M)`	Submatrix 1-row, IDX2-col for a piece matrix M of MM
`MM(IDX1,1,M)`	Submatrix IDX1-row, 1-col for a piece

matrix M of MM

`M1(IDX2)`	`= M2;`	Set M2 at IDX2-elem of M1
`M1(1,IDX2)`	`= M2;`	Set M2 at 1-row, IDX2-col of M1
`M1(IDX1,IDX2)`	`= M2;`	Set M2 at IDX1-row, IDX2-col of M1
`M1(IDX1,1)`	`= M2;`	Set M2 at IDX1-row, 1-col of M1
`M1(IDX1,1:2)`	`= M2;`	Set M2 at IDX1-row, 1st-2nd-col of M1
`M1(1:2,IDX2)`	`= M2;`	Set M2 at 1st-2nd-row, IDX2 of M1
`MM(IDX2,M)`	`= M2;`	Set M2 at IDX2-elem for a piece matrix M of M1
`MM(IDX1,IDX2,M)`	`= M2;`	Set M2 at IDX1-row, IDX2-col for a piece matrix M of M1
`MM(1,IDX2,M)`	`= M2;`	Set M2 at 1-row, IDX2-col for a piece matrix M of M1
`MM(IDX1,1,M)`	`= M2;`	Set M2 at IDX1-row, 1-col for a piece matrix M of M1

15.5.6 Piece-Matrix (Array of Matrix)

Matrices may be considered as 1-dimensional or 2-dimensional arrays of a certain object, such as real numbers, complex number, polynomials, and rational functions, while arrays of matrix can be represented by introducing the piece-matrix notion. The piece-matrix is used to specify the unit size of arrays of matrix. List class is available to describe an array which contains various size matrices. For example,

```
A = [[1 2][3 4]];
AA = Z(2, 3, A);
```

lead to a 4×6 zero matrix `AA`. And

```
AA(0, 0, A) = [[1 2][3 4]];
AA(1, 2, A) = [[5 6][7 8]];
```

result in

$$\texttt{AA} = \begin{bmatrix} 1 & 2 & \texttt{Z(2)} & \texttt{Z(2)} \\ 3 & 4 & & \\ \texttt{Z(2)} & \texttt{Z(2)} & 5 & 6 \\ & & 7 & 8 \end{bmatrix}$$

where matrix `A` is called **piece-matrix** of matrix `AA`, and the index of arrays of matrix start from 0. Using another piece-matrix `B (= [1])` instead of $\hat{\texttt{A}}$ and

```
B = [1];
AA(0, 0, B) = [[1 2][3 4]];
AA(1, 2, B) = [[5 6][7 8]];
```

yield another result:

$$\texttt{AA} = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 4 & 5 & 6 & 0 & 0 \\ 0 & 0 & 7 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

15.6 Comparison of Matrices and Arrays

We can check the positivity of a matrix and compare two matrices by using relational operators `>`, `>=`, `<`, `<=`. If an array is compared, the comparison is done element-by-element and the result is an array with (0—1) elements. This result can be used for the argument of the functions, `all()` and `any()`.

`Matrix < 0`	`maxeigval(Matrix) < 0`
`Matrix <= 0`	`maxeigval(Matrix) <= 0`
`Matrix > 0`	`mineigval(Matrix) > 0`
`Matrix >= 0`	`mineigval(Matrix) >= 0`
`Matrix == 0`	`Matrix is a Zero Matrix`
`Matrix != 0`	`Matrix is not a Zero Matrix`
`0 < Matrix`	`mineigval(Matrix) > 0`
`0 <= Matrix`	`mineigval(Matrix) >= 0`

`0 > Matrix`	`maxeigval(Matrix) < 0`
`0 >= Matrix`	`maxeigval(Matrix) <= 0`
`0 == Matrix`	`Matrix is a Zero Matrix`
`0 != Matrix`	`Matrix is not a Zero Matrix`
`Matrix1 < Matrix2`	`maxeigval(Matrix1 - Matrix2) < 0`
`Matrix1 <= Matrix2`	`maxeigval(Matrix1 - Matrix2) <= 0`
`Matrix1 > Matrix2`	`mineigval(Matrix1 - Matrix2) > 0`
`Matrix1 >= Matrix2`	`mineigval(Matrix1 - Matrix2) >= 0`
`Matrix1 == Matrix2`	`Matrix1 is equal to Matrix2`
`Matrix1 != Matrix2`	`Matrix1 is not equal to Matrix2`

`Array1 < Array2`	`Element-by-element relation:`
`Array1 > Array2`	`Element-by-element relation:` `(0|1)-Array`
`Array1 <= Array2`	`Element-by-element relation:` `(0|1)-Array`
`Array1 >= Array2`	`Element-by-element relation:` `(0|1)-Array`
`Array1 == Array2`	`Element-by-element relation:` `(0|1)-Array`
`Array1 != Array2`	`Element-by-element relation:` `(0|1)-Array`
`(Int|Real-Num)< Array`	`Element-by-element relation:`
`(Int|Real-Num)> Array`	`Element-by-element relation:` `(0|1)-Array`
`(Int|Real-Num)<= Array`	`Element-by-element relation:` `(0|1)-Array`
`(Int|Real-Num)>= Array`	`Element-by-element relation:` `(0|1)-Array`
`(Int|Real-Num)== Array`	`Element-by-element relation:` `(0|1)-Array`
`(Int|Real-Num)!= Array`	`Element-by-element relation:` `(0|1)-Array`
`Array1 < (Int|Real-Num)`	`Element-by-element relation:`
`Array1 > (Int|Real-Num)`	`Element-by-element relation:` `(0|1)-Array`
`Array1 <= (Int|Real-Num)`	`Element-by-element relation:`

```
                              (0|1)-Array
Array1 >= (Int|Real-Num)      Element-by-element relation:
                              (0|1)-Array
Array1 == (Int|Real-Num)      Element-by-element relation:
                              (0|1)-Array
Array1 != (Int|Real-Num)      Element-by-element relation:
                              (0|1)-Array


Array1 && Array2      Element-by-element logical operation:
                      (0|1)-Array
Array1 || Array2      Element-by-element logical operation:
                      (0|1)-Array
!  Array1             Element-by-element logical operation:
                      (0|1)-Array
```

15.7 Matrix Functions

MATX provides the following matrix functions. The algorithms in EISPACK [12], [11], [2] is used for eigenvalue problems. . The algorithms in [10] is used for solving ordinary differential equations according to 4th order Runge-Kutta methods automatically changing the step to guarantee the prespecified error. The reference [1] is used for solving ordinary differential equations according to RKF45 methods with the prespecified step. LU decomposition, QR decomposition, and QZ decomposition are implemented by refering [3].

```
Integer  rank(Matrix M1);           Rank
Integer  rank(Matrix M1, Real tol); Rank
                                    tol: Tolerance
Integer  Rows(Matrix M1);           Row number
Integer  Cols(Matrix M1);           Column number
Integer  length(Matrix M1);         Length of row
                                    or column vector
Integer  isreal(Matrix M1);         Is real matrix ?
                                    (True = 1, False = 0)
Integer  iscomplex(Matrix M1);      Is complex matrix ?
```



```
                                    (True = 1, False = 0)
Integer  isscalar(Matrix M1);       Is scalar value ?
                                    (True = 1, False = 0)
Array    finite(Matrix M1);         ones where the elements of M1
                                    are finite and zeros where
                                    they are infinite or NaN.
Array    isnan(Matrix M1);          ones where the elements are NaN
                                    zeros where they are not.
Matrix   sgn(Matrix M1);            Element-by-element sgn()


Real max(Matrix M1);          Maximum element
Real min(Matrix M1);          Minimum element
List maximum(Matrix M1);      Maximum element and its location
                              {r1, i, j} = maximum(M1);
                              r1 == M1(i,j)
List minimum(Matrix M1);      Minimum element and its location
                              {r1, i, j} = maximum(M1);
                               r1 == M1(i,j)
Real maxsing(Matrix M1);      Maximum singular value
Real minsing(Matrix M1);      Minimum singular value
Real det(Matrix M1);          Determinant
Real sum(Matrix M1);          Sum of all elements
Real prod(Matrix M1);         Product of all elements
Real mean(Matrix M1);         Mean of all elements
Real trace(Matrix M1);        Trace of real matrix
Real frobnorm(Matrix M1);     Frobenius norm
                              (Euclidean norm)
Real infnorm(Matrix M1);      Infinity norm
Real norm(Matrix M1, Integer p);
                                   M1 : Matrix
                                   p=1 1-norm
                                   p=2 maximum singular value
                                   M1 : Vector p-norm
Real norm(Matrix M1);              norm(Matrix M1, 2)


Complex max(Matrix M1);     Maximum element of a complex matrix
Complex min(Matrix M1);     Minimum element of a complex matrix
```

```
List maximum(Matrix M1);    Maximum element and its location
                            {c1, i, j} = maximum(M1);
                            c1 == M1(i,j)
List minimum(Matrix M1);    Minimum element and its location
                            {c1, i, j} = maximum(M1);
                            c1 == M1(i,j)
Complex det(CMatrix CM1);   Determinant
Complex trace(CMatrix CM1); Trace
Complex sum(CMatrix CM1);   Sum of all elements
Complex prod(Matrix M1);    Product of all elements
Complex mean(CMatrix CM1);  Mean of all elements


Polynomial det(PMatrix PM1);      Determinant
Polynomial trace(PMatrix PM1);    Trace
Polynomial sum(PMatrix PM1);      Sum of all elements
Polynomial prod(Matrix M1);       Product of all elements
Polynomial mean(PMatrix PM1);     Mean of all elements
Polynomial frobnorm(PMatrix PM1); Frobenius norm


Rational  det(RMatrix RM1);       Determinant
Rational  trace(RMatrix RM1);     Trace
Rational  sum(RMatrix RM1);       Sum of all elements
Rational  prod(Matrix M1);        Product of all elements
Rational  mean(RMatrix RM1);      Mean of all elements
Rational  frobnorm(RMatrix RM1);  Frobenius norm


Matrix eigval(Matrix M1);            Eigenvalues
Matrix eigval(Matrix M1, Matrix M2); Generalized eigenvalues
Matrix eigvec(Matrix M1);            Eigenvectors
Matrix eigvec(Matrix M1, Matrix M2); Generalized eigenvectors


Matrix  singval(Matrix M1);      Singular values
Matrix  singleftvec(Matrix M1);  Left singular vectors
Matrix  singrightvec(Matrix M1); Right singular vectors
```



```
Matrix  vectorprod(Matrix M1, Matrix M2); Vector product
Matrix  exp(Matrix M1);        Exponential function
Matrix  sqrt(Matrix M1);       Square root function
Matrix  log(Matrix M1);        Natural logarithm function
Matrix  kernel(Matrix M1);     Kernel (Null space)
Matrix  conj(Matrix M1);       Complex conjugate
Matrix  pseudoinv(Matrix M1);  Pseudoinverse
Matrix  pseudoinv(Matrix M1, Real tol);  Pseudoinverse
                               tol: Tolerance


Matrix  vec2diag(Matrix M1);   Col-vector to diagonal matrix
Matrix  vec2diag(Matrix M1, Integer k);
                              Col-vector to diagonal matrix
                              (K > 0 is above the main diagonal)
                              (K < 0 is below the main diagonal)
Matrix  diag2vec(Matrix M1);   Diagonal matrix to col-vector
Matrix  diag2vec(Matrix M1, Integer k);
                              Diagonal matrix to col-vector
                              (K > 0 is above the main diagonal)
                              (K < 0 is below the main diagonal)
Matrix  fliplr(Matrix M1);     Flip matrices left-right
Matrix  flipud(Matrix M1);     Flip matrices up-down.
List    sort(Matrix M1);       Sorting in ascending order.
                               For complex matrices,
                               sorted by abs(M1)
      {M2, idx} = sort(M1);    M2:  Result matrix
                               idx: Indices used in the sort.
                               M2 = M1(idx)
List    sort(Array A1);        Sorting in ascending order.
                               For complex matrices,
                               sorted by abs(A1)
      {A2, idx} = sort(A1);    A2:  Result array
                               idx: Indices used in the sort.
                               A2 = A1(idx)
Matrix max(Matrix M1, Matrix M2); A matrix the same size as M1
                                  and M2 with the largest elem-
                                  ents taken from M1 or M2
```

```
Matrix min(Matrix M1, Matrix M2); A matrix the same size as M1
                                  and M2 with the smallest elem-
                                  ents taken from M1 or M2
Array rem(Array A1, Real r1);  rem(,r1) for echo element of A1
Array rem(Array A1, Array A2); rem() for echo element of A1
                               and element of A2


Matrix  rand(Integer m);   m x m Uniformly distributed matrix
Matrix  rand(Integer m, Integer n);
                           m x n Uniformly distributed matrix
Matrix  rand(Matrix A);    Uniformly distributed matrix
                           as large as matrix A
Matrix  randn(Integer m);  m x m Normaly distributed matrix
Matrix  randn(Integer m, Integer n);
                           m x n Normaly distributed matrix
Matrix  randn(Matrix A);   Normaly distributed matrix
                           as large as matrix A


Matrix  rngkut4(Real t, Matrix x, void dx())
                : dt = 1.0E-7, u = Z(0);
Matrix  rngkut4(Real t, Matrix x, void dx(), Matrix u)
                : dt = 1.0E-7
Matrix  rngkut4(Real t, Matrix x, void dx(), Matrix u,
                Real dt)
        To solve ordinary differential equations according
        to 4th order Runge-Kutta methods.


List Ode(Real t0, Real t1, Matrix x, void dx());
              : h=4.0E-2, dtsav=1.0E-3
List Ode(Real t0, Real t1, Matrix x, void dx(),
              void input()); : h=4.0E-2, dtsav=1.0E-3
List Ode(Real t0, Real t1, Matrix x, void dx(),
              void input(), Real h); : dtsav=1.0E-3
List Ode(Real t0, Real t1, Matrix x, void dx(),
              void input(), Real h, Real dtsav);
     To solve ordinary differential equations according
```



```
     to 4th order Runge-Kutta methods with the prespecified
     step 'h' between [t0 t1].
     {T,X,U} = Ode();


List OdeAuto(Real t0, Real t1, Matrix x, void dx());
               : eps=1.0E-6, dtsav=1.0E-3,
                 dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeAuto(Real t0, Real t1, Matrix x, void dx(),
             void input());
              : eps=1.0E-6, dtsav=1.0E-3
                dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeAuto(Real t0, Real t1, Matrix x, void dx(),
             void input(), Real eps);
             : dtsav=1.0E-3, dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeAuto(Real t0, Real t1, Matrix x, void dx(),
             void input(), Real eps, Real dtsav);
             : dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeAuto(Real t0, Real t1, Matrix x, void dx(),
             void input(), Real eps,
             Real dtsav, Real dtmin);
             : dtmax=abs(t1-t0)/5
List OdeAuto(Real t0, Real t1, Matrix x, void dx(),
             void input(), Real eps,
             Real dtsav, Real dtmin, Real dtmax);
     To solve ordinary differential equations according to
     4th order Runge-Kutta methods automatically changing
     the step to guarantee the prespecified error between
     [t0 t1], where the minimum and maximum step are
     dtmin and dtmax, respectively.
     {T,X,U} = OdeAuto();


List OdeHybrid(Real t0, Real t1, Real dt, Matrix x,
         void dx())
List OdeHybrid(Real t0, Real t1, Real dt, Matrix x,
         void dx(), void input()) : h=4.0E-2, dtsav=1.0E-3
List OdeHybrid(Real t0, Real t1, Real dt, Matrix x,
         void dx(), void input(), Real h) : dtsav=1.0E-3
```

```
List OdeHybrid(Real t0, Real t1, Real dt, Matrix x,
          void dx(), void input(), Real h, Real dtsav)
     To solve ordinary differential equations according to
     4th order Runge-Kutta methods with the prespecified
     step 'h' between [t0 t1], where the external input
     is constant for the prespecified sampling interval.
     If the step 'h' is not prespecified and 4.0E-2 < h,
     h is equal to dt.
     {T,X,U} = OdeHybrid();

List OdeHybridAuto(Real t0, Real t1, Real dt, Matrix x,
                   void dx())
                   : eps=1.0E-6, dtsav=1.0E-3,
                     dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeHybridAuto(Real t0, Real t1, Real dt, Matrix x,
                    void dx(), void input())
                   : eps=1.0E-6, dtsav=1.0E-3
                     dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeHybridAuto(Real t0, Real t1, Real dt, Matrix x,
                    void dx(), void input(), Real eps)
                : dtsav=1.0E-3, dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeHybridAuto(Real t0, Real t1, Real dt, Matrix x,
              void dx(), void input(), Real eps, Real dtsav)
                   : dtmin=EPS, dtmax=abs(t1-t0)/5
List OdeHybridAuto(Real t0, Real t1, Real dt, Matrix x,
                    void dx(), void input(), Real eps,
                    Real dtsav, Real dtmin)
                   : dtmax=abs(t1-t0)/5
List OdeHybridAuto(Real t0, Real t1, Real dt, Matrix x,
                    void dx(), void input(), Real eps,
                    Real dtsav, Real dtmin, Real dtmax)
     To solve ordinary differential equations according to
     4th order Runge-Kutta methods automatically changing
     the step to guarantee the prespecified error between
     [t0 t1], where the external input is constant
     for the prespecified sampling interval.
     And the minimum and maximum step are dtmin and dtmax.
     {T,X,U} = OdeHybridAuto();
```



```
Matrix  rkf45(Real t, Matrix x, void dx())
                : dt = 1.0E-7, u = Z(0);
Matrix  rkf45(Real t, Matrix x, void dx(), Matrix u)
                : dt = 1.0E-7
Matrix  rkf45(Real t, Matrix x, void dx(), Matrix u, Real dt)
        To solve ordinary differential equations according to
        RKF45 methods.


List Ode45(Real t0, Real t1, Matrix x, void dx());
             : h=4.0E-2, dtsav=1.0E-3
List Ode45(Real t0, Real t1, Matrix x, void dx(),
             void input()); : h=4.0E-2, dtsav=1.0E-3
List Ode45(Real t0, Real t1, Matrix x, void dx(),
             void input(), Real h); : dtsav=1.0E-3
List Ode45(Real t0, Real t1, Matrix x, void dx(),
             void input(), Real h, Real dtsav);
     To solve ordinary differential equations according to
     RKF45 methods with the prespecified
     step 'h' between [t0 t1].
     {T,X,U} = Ode45();


List Ode45Auto(Real t0, Real t1, Matrix x, void dx());
             : eps=1.0E-6, dtsav=1.0E-3,
               dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45Auto(Real t0, Real t1, Matrix x, void dx(),
              void input());
             : eps=1.0E-6, dtsav=1.0E-3,
               dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45Auto(Real t0, Real t1, Matrix x, void dx(),
              void input(), Real eps);
             : dtsav=1.0E-3, dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45Auto(Real t0, Real t1, Matrix x, void dx(),
              void input(), Real eps, Real dtsav);
             : dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45Auto(Real t0, Real t1, Matrix x, void dx(),
              void input(), Real eps,
              Real dtsav, Real dtmin);
```

```
              : dtmax=abs(t1-t0)/5
List Ode45Auto(Real t0, Real t1, Matrix x, void dx(),
               void input(), Real eps,
               Real dtsav, Real dtmin, Real dtmax);
     To solve ordinary differential equations according to
     RKF45 methods automatically changing
     the step to guarantee the prespecified error between
     [t0 t1], where the minimum and maximum step are
     dtmin and dtmax, respectively.
     {T,X,U} = Ode45Auto();


List Ode45Hybrid(Real t0, Real t1, Real dt, Matrix x,
         void dx())
List Ode45Hybrid(Real t0, Real t1, Real dt, Matrix x,
         void dx(), void input()) : h=4.0E-2, dtsav=1.0E-3
List Ode45Hybrid(Real t0, Real t1, Real dt, Matrix x,
         void dx(), void input(), Real h) : dtsav=1.0E-3
List Ode45Hybrid(Real t0, Real t1, Real dt, Matrix x,
          void dx(), void input(), Real h, Real dtsav)
     To solve ordinary differential equations according to
     RKF45 methods with the prespecified
     step 'h' between [t0 t1], where the external input
     is constant for the prespecified sampling interval.
     If the step 'h' is not prespecified and 4.0E-2 < h,
     h is equal to dt.
     {T,X,U} = Ode45Hybrid();


List Ode45HybridAuto(Real t0, Real t1, Real dt, Matrix x,
                     void dx())
                     : eps=1.0E-6, dtsav=1.0E-3,
                       dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45HybridAuto(Real t0, Real t1, Real dt, Matrix x,
                     void dx(), void input())
                     : eps=1.0E-6, dtsav=1.0E-3
                       dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45HybridAuto(Real t0, Real t1, Real dt, Matrix x,
                     void dx(), void input(), Real eps)
```



```
                     : dtsav=1.0E-3,
                       dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45HybridAuto(Real t0, Real t1, Real dt, Matrix x,
             void dx(), void input(), Real eps, Real dtsav)
             : dtmin=EPS, dtmax=abs(t1-t0)/5
List Ode45HybridAuto(Real t0, Real t1, Real dt, Matrix x,
             void dx(), void input(), Real eps,
             Real dtsav, Real dtmin)
             : dtmax=abs(t1-t0)/5
List Ode45HybridAuto(Real t0, Real t1, Real dt, Matrix x,
             void dx(), void input(), Real eps,
             Real dtsav, Real dtmin, Real dtmax)
     To solve ordinary differential equations according to
     RKF45 methods automatically changing
     the step to guarantee the prespecified error between
     [t0 t1], where the external input is constant
     for the prespecified sampling interval.
     And the minimum and maximum step are dtmin and dtmax.
     {T,X,U} = Ode45HybridAuto();


void OdeStop();       Stop the execution of functions:
                      Ode(), OdeAuto(), OdeHybrid(),
                      OdeHybridAuto(), Ode45(), Ode45Auto(),
                      Ode45Hybrid(), Ode45HybridAuto()


List size(Matrix M1);  Row number and column number
                       {row, col} = size(A);
List eig(Matrix M1);   Eigen Problem
                       {val, vec} = eig(A);
                       A * vec = vec * val;
                       val: Eigenvalues(diagonal matrix)
                       vec: Eigenvectors
List eig(Matrix M1, Matrix M2);
                       Generalized Eigen Problem
                       {val, vec} = eig(A, B);
                       A * vec = B * vec * val;
                       val: Generalized eigenvalues
```

```
                        (diagonal matrix)
                    vec: Generalized eigenvectors


List svd(Matrix M1);    Singular Value Decomposition
                        {U, D, V} = svd(A);
                        U * D * V# = A;
                        U: Left Singular vectors
                        V: Right Singular vectors
                        D: Singular Values
                           (same size as A)
List lu(Matrix M1);     LU Decomposition
                        {L, U, P} = lu(A);
                        L * U = P * A;
                        L: Lower triangular matrix,
                        U: Upper triangular matrix,
                        P: Permutation matrix
List hess(Matrix M1);   Hessenberg Decomposition
                        {Q, H} = hess(A);
                        Q# * A * Q = H;
                        Q: Orthogonal or unitary matrix
                        H: Hessenberg matrix
List schur(Matrix M1); Schur Decomposition
                        {U, T} = schur(A);
                        U# * A * U = T;
                        U: Orthogonal or unitary matrix
                        T: Pseudo-upper triangular or
                           upper triangular matrix


List qr(Matrix M1);     QR decomposition
                        {Q, R} = qr(A);
                        Q * R = A;
                        Q: Orthogonal or unitary matrix
                        R: Upper triangular matrix
List qr_p(Matrix M1);   QR decomposition (with permutation)
                        {Q, R, P} = qr_p(A);
                        Q * R = A * P;
                        Q: Orthogonal or unitary matrix
```



```
                        R: Upper triangular matrix
                        P: Permutation matrix
List qz(Matrix M1, Matrix M2);
                        QZ decomposition
                        {AA, BB, Q, Z, VEC} = qz(A,B)
                        Q# * A * Z = AA;
                        Q# * B * Z = BB;
                        val = diag2vec(AA) ./ diag2vec(BB);
                        A * VEC = B * VEC * vec2diag(val);
                       AA: Upper triangular matrix
                       BB: Upper triangular matrix
                        Q: Orthogonal or unitary matrix
                        Z: Orthogonal or unitary matrix
                      VEC: Generalized eigenvectors
                      val: Generalized eigenvalues
```

15.8 Array Functions

MATX provides the following array functions.

```
Integer Rows(Array A1);       Row number
Integer Cols(Array A1);       Column number
Integer isreal(Array A1);     Is real ?
                              (True = 1, False = 0)
Integer  iscomplex(Array A1); Is complex ?
                              (True = 1, False = 0)
Integer  all(Array A1);       All elements nonzero ?
                              (True = 1, False = 0)
Integer  all(Matrix M1);      All elements nonzero ?
                              (True = 1, False = 0)
Integer  any(Array A1);       Any nonzero elements ?
                              (True = 1, False = 0)
Integer  any(Matrix M1);      Any nonzero elements ?
                              (True = 1, False = 0)
Array    finite(Array A1);    ones where the elements of M1
                              are finite and zeros where
                              they are infinite or NaN.
```

```
Array    isnan(Array A1);     ones where the elements are
                              NaN zeros where they are not.

Index find(Array A1);        Index of nonzero elements
                             If A1 is not a vector, the
                             elements are checked row-wise.
Index find(Matrix M1);       Index of nonzero elements
                             If M1 is not a vector, the
                             elements are checked row-wise.


Real max(Array A1);              Maximum element
Real min(Array A1);              Minimum element
Real frobnorm(Array A1);         Frobenius norm
Real sum(Array A1);              Sum of all elements
Real mean(Array A1);             Mean of all elements


Complex sum(CArray CA1);         Sum of all elements
Complex mean(CArray CA1);        Mean of all elements


Polynomial frobnorm(PArray PA1); Frobenius norm
Polynomial sum(PArray PA1);      Sum of all elements
Polynomial mean(PArray PA1);     Mean of all elements


Rational frobnorm(RArray RA1);   Frobenius norm
Rational sum(RArray RA1);        Sum of all elements
Rational mean(RArray RA1);       Mean of all elements


Array logspace(Real d1, Real d2)
              Logarithmically spaced vector (split=50)
Array logspace(Real d1, Real d2, Integer split)
              Logarithmically spaced vector
Array linspace(Real d1, Real d2)
              Linearly spaced vector (split=100)
Array linspace(Real d1, Real d2, Integer split)
              Linearly spaced vector
```



```
Array  vec2diag(Array M1);   Col-vector to diagonal array
Array  vec2diag(Array M1, Integer k);
                             Col-vector to diagonal array
                             (K > 0 is above the main diagonal)
                             (K < 0 is below the main diagonal)
Array  diag2vec(Array M1);   Diagonal array to col-vector
Array  diag2vec(Array M1, Integer k);
                             Diagonal array to col-vector
                             (K > 0 is above the main diagonal)
                             (K < 0 is below the main diagonal)
Array  fliplr(Array M1);     Flip arrays left-right
Array  flipud(Array M1);     Flip arrays up-down.
List   sort(Array M1);       Sorting in ascending order.
                             For complex arrays,
                             sorted by abs(A1)
      {A2, idx} = sort(A1);  A2:  Result array
                             idx: Indices used in the sort.
                             A2 = A1(idx)
List size(Array A1);         Row number and column number
                             {row, col} = size(A);
Array   rand(Array A);       Uniformly distributed array
                             as large as array A
Array   randn(Array A);      Normaly distributed  array
                             as large as array A
Array max(Array A1, Array A2); An array the same size as A1
                               and A2 with the largest elem-
                               ents taken from A1 or A2
Array min(Array A1, Array A2); An array the same size as A1
                               and A2 with the smallest elem-
                               ents taken from A1 or A2


Array fft(Array data);             Fast Fourie Transforms
Array fft(Array data, Integer n);  Fast Fourie Transforms
                                   n = 2^m
Array ifft(Array data);            Inverse Fast Fourie
                                   Transforms
Array ifft(Array data, Integer n); Inverse Fast Fourie
```

```
                                Transforms
                                n = 2^m


Array sgn(Array A1)         Element-by-element sgn()
Array abs(Array A1);        Element-by-element abs()
Array arg(Array A1);        Element-by-element arg()
Array exp(Array A1);        Element-by-element exp()
Array sqrt(Array A1);       Element-by-element sqrt()
Array log(Array A1);        Element-by-element log()
Array log10(Array A1);      Element-by-element log10()
Array conj(Array A1);       Element-by-element conj()
Array sin(Array A1);        Element-by-element sin()
Array asin(Array A1);       Element-by-element asin()
Array sinh(Array A1);       Element-by-element sinh()
Array asinh(Array A1);      Element-by-element asinh()
Array cos(Array A1);        Element-by-element cos()
Array acos(Array A1);       Element-by-element acos()
Array cosh(Array A1);       Element-by-element cosh()
Array acosh(Array A1);      Element-by-element cosh()
Array tan(Array A1);        Element-by-element tan()
Array atan(Array A1);       Element-by-element atan()
Array tanh(Array A1);       Element-by-element tanh()
Array atanh(Array A1);      Element-by-element atanh()
Array ceil(Array A1);       Element-by-element ceil()
Array floor(Array A1);      Element-by-element floor()
Array fix(Array A1);        Element-by-element fix()
Array round(Array A1);      Element-by-element round()
```

Chapter 16

List

Declaration:

```
List a, b, c;
```

Example:

```
{4, (3,4), [1 2]}, {"Hello", s+1, (s+1)/(s+2)}
```

MATX has a list data type. The list is a collection of data objects not necessarily of the same type. A list can contain any of the MATX data types as well as other lists. List enables us to define multiple-output functions.

16.1 How To Describe Lists

The method of describing a list class object is to use an explicit list of objects which are separated by the commas ",," and surrounded by a couple of braces, "{" and "}". For example, entering the statement

```
x = {3, (3,4), [1 2]};
```

results in a list which contains an integer, a complex-number, and a real matrix. List elements can be any MATX expressions. For example, the statement


```
x = {sqrt(3.0), ((1.0+2.0)*4.0,:), [1 2] + [3 4]};
```

produces a list which contains a real-number, complex-number, and a real-matrix. The function "`makelist()`" makes a list which contains arbitrary number of elements. The elements are undefined.

```
x    = makelist(3);
```

16.2 List Elements

Individual list elements can be specified with the index and the class name of the element separated by a comma inside parentheses "(" and ")". (Index begins from 1.)

```
x = {sqrt(3.0), ((1.0+2.0)*4.0,:), [1 2] + [3 4]};
a = x(1, Real);     /* a = 1.732 */
b = x(2, Complex);  /* b = (12,0) */
```

Any data can be assigned to an element of the list which is specified with the index inside parentheses on the left-hand side. (Index begins from 1.) And, a list on the right-hand side can be assigned to the position which is specified with an interval inside parentheses on the left-hand side. The length of the list on the right-hand side must be more than or equal to the length of the specified interval. If the length of the list on the right-hand side is less than that of the specified interval, only the first corresponding elements are assigned to the list on the left-hand side and the others are discarded.

```
x = makelist(4);
x(1) = (3,4);
a = x(1, Complex);  /* a = (3,4) */
x(2:4) = {sqrt(3.0), (5,6), [1 2] + [3 4], "Hello"};
/* x = {(3,4), 1.732, (5,6), [4 6]} */
```

16.3 Append a New Data

The function "`appendlist()`" is used in order to append a new data to the list.

```
x = {sqrt(3.0), ((1.0+2.0)*4.0,:)};
appendlist(x, "Hello");
/* x --> {1.7320508, (12, 0), "Hello"} */
```

16.4 Attachment and Extraction

Long lists can be produced from several short lists. For example, we can make a long list `z` from two short lists `x` and `y` as follows.

```
x = {4, (3,4), [1 2]};
y = {"Hello", 2.0};
z = {x, y};
/* z = {{4, (3,4), [1 2]}, {"Hello", 2.0}} */
```

Any part of a list can be extracted by specifying the interval as two indices. For example,

```
x2 = x(1:2);  /* x2 = {4, (3,4)} */
```

extracts the first 2 elements from `x`.

16.5 List Functions

MATX provides the following list functions.

```
Integer length(List L1);               Length of the list
List    appendlist(List L1, Object A); Append a data to the list
```

Chapter 17

Examples

There are several illustrative example programs in this chapter, such as a program to solve continuous-time riccati equations, a program to solve discrete-time riccati equations, a program to obtain the transfer functions and transfer function matrices of linear systems, a program to calculate the bode and nyquist frequency responses, and three simulation programs of the state variable feedback control and the dynamic feedback control.

17.1 Continuous-Time Riccati Equations

The solution of riccati equation of continuous-time linear systems can be obtained by solving the eigenvalue problem of a Hamilton matrix [4]. Since the function `eigvec()` returns eigenvectors ordered according to the magnitude of the real-part of the eigenvalues, the statements on the 20th and 21st line extract the eigenvectors corresponding to the stable eigenvalues from the matrix `vec`. The following statements are explanations of the list.

- On the 1st and 11th line, beginning of functions.
- On the 3rd and 14th, declaration of local variables.
- On the 6th line, reading matrices from standard input.
- On the 7th line, call the function `CRiccati()`.
- On the 8th line, displaying the solution matrix `P`.
- On the 12th line, declaration of arguments of the function.
- On the 16th line, making a Hamilton matrix [4].
- On the 19th line, calculation of eigenvectors.
- On the 20th and 21st line, partitioning matrix `vec` using a piece-matrix `A`.

```
   Func void main()
   {
     Matrix A, B, Q, R, P;  /* Declaration of variables */
     Matrix CRiccati();     /* Declaration of function  */
05:
     read A, B, Q, R;           /* Read matrices   */
     P = CRiccati(A, B, Q, R); /* Call CRiccati() */
     print P;                   /* Print out P     */
   }
10:
   Func Matrix CRiccati(A, B, Q, R)
     Matrix A, B, Q, R;
   {
     Matrix H, U, V, vec, P;
15:
     H = [[ A   , -B*R~*B#]   /* Make a Hamilton matrix */
          [-Q   , -A#     ]];

     vec = eigvec(H);  /* Eigenvectors of H.         */
20:  U = vec(0, 1, A); /* Extract the eigenvectors   */
     V = vec(1, 1, A); /* correspoding to the stable */
                       /* eigenvalues.               */
     if (isreal(A) && isreal(B) &&
         isreal(Q) && isreal(R)) {
         P = Re(V*U~);
25:  } else {
         P = V*U~;
     }

     return P;
30:}
```

List 1: The Program of Continuous-Time Riccati Equation

17.2 Discrete-Time Riccati Equations

The following list is the program which obtains the solution of the riccati equation of a discrete-time linear system by iteration. The solution `P` and a temporary matrix `PP` are initialized with identity matrix which is the same size as matrix `A`. In order to reduce the calculation error, matrix `P` is symmetrized. The calculation of the discrete-time riccati equation is iterated until the frobenius norm of the difference between matrix `P` and the previous solution matrix `PP` is less than `eps`. The following statements are explanations of the list.

- On the 1st and 16th line, beginning of functions.
- On the 3rd, 19th, and 20th line, declaration of local variables.
- On the 6th to 8th, 10th, and 23rd line, entering constant matrices.
- On the 9th, 26th, and 27th line, calculation of matrices.
- On the 12th line, calling the function `DRiccati()`.
- On the 13th line, displaying the solution matrix `P`.
- On the 17th line, declaration of arguments of the function.
- On the 26th line, examination of frobenius norm of "`P - PP`".

```
   Func void main()
   {
     Matrix A, B, C, Q, R, P, PP;
     Matrix DRiccati();
05:
     A = [[0 1][-3, 4]]; /* System Parameters. */
     B = [0 1]';
     C = [-2, 1];
     Q = C'*C;
10:  R = I(1);

     P = DRiccati(A, B, Q, R); /* Call DRiccati(). */
     print P;                  /* Print P.         */
   }
15:
   Func Matrix DRiccati(A, B, Q, R)
     Matrix A, B, Q, R;
   {
     Matrix P, PP;
20:  Real eps;

     P = PP = I(A); /* Initialization of the solution. */
     eps = 1.0E-7;

25:  /* Repeat the loop until the solution converges. */
     while (frobnorm(P - PP) > eps) {
         PP = P;
         P = Q + A'*P*A - A'*P*B*(R + B'*P*B)~ * B'*P*A;
         P = (P' + P)/2.0;   /* Symmetrise matrix P. */
30:  }

     return P;  /* Return the solution. */
   }
```

List 2: The Program of Discrete-Time Riccati Equation

17.3 Transfer Function

The transfer function of a linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

is calculated according to Faddev's algorithm as follows.

$$\begin{aligned} G(s) &= C(sI - A)^{-1}B \\ &= C\frac{1}{\det(sI - A)}(\Gamma_{n-1}s^{n-1} + \Gamma_{n-2}s^{n-2} + \cdots + \Gamma_0)B \end{aligned}$$

where

$$\det(sI - A) = s^n + \alpha_{n-1}s^{n-1} + \alpha_{n-2}s^{n-2} + \cdots + \alpha_0$$

and

$$\begin{array}{ll} \Gamma_{n-1} = I & \alpha_{n-1} = -\mathrm{tr}(A\Gamma_{n-1}) \\ \Gamma_{n-2} = A\Gamma_{n-1} + a_{n-1}I & \alpha_{n-2} = -\mathrm{tr}(A\Gamma_{n-2})/2 \\ \Gamma_{n-3} = A\Gamma_{n-2} + a_{n-2}I & \alpha_{n-3} = -\mathrm{tr}(A\Gamma_{n-3})/3 \\ \cdots\ \cdots\ \cdots & \cdots\ \cdots\ \cdots \\ \Gamma_i = A\Gamma_{i+1} + \alpha_{i+1}I & \alpha_i = -\mathrm{tr}(A\Gamma_i)/(n-i) \\ \cdots\ \cdots\ \cdots & \cdots\ \cdots\ \cdots \\ \Gamma_0 = A\Gamma_1 + \alpha_1 I & \alpha_0 = -\mathrm{tr}(A\Gamma_0)/n \end{array}$$

Here, $\mathrm{tr}(X)$ denotes the sum of diagonal elements of X.

```
   Func void main()
   {
     Matrix A, B, C, G;    /* Declaration of variables. */
     Matrix Faddeev();
05:
     read A, B, C;         /* Read matrices  */
     G = Faddeev(A, B, C); /* Call Faddeev() */
     print G;              /* Print out G    */
   }
10:
   Func Matrix Faddeev(A, B, C)
     Matrix A, B, C;
   {
     Matrix Gamma, N;
15:  Polynomial s, ch;
     Integer i, n;

     s = $;         /* Polynomial variable */
     n = Rows(A);   /* System order        */
10:
     Gamma = Z(n,A);
     ch = Polynomial(Z(1,n+1)); /* Characteristic
                                   polynomial        */
     ch(n) = 1.0;               /* Monic polynomial  */
25:
     Gamma(n-1,A) = I(n);
     ch(n-1) = - trace(A);

     for (i = n-2; i >= 0; i--) {
30:      Gamma(i,A) = A*Gamma(i+1,A) + ch(i+1)*I(n);
         ch(i) = - trace(A*Gamma(i,A)) / (n - i);
     }

     N = Z(Rows(C), Cols(B));
35:  for (i = 0; i < n; i++) {
         N = N + C*Gamma(i,A)*B * s^i;
     }

     return N / ch;  /* Return the transfer function. */
40:}
```

List 3: The Program of the Transfer Function

17.4 Bode and Nyquist Frequency Response

The following program involves calculation of Bode frequency response and Nyquist frequency response. The following statements are explanations of the list.

- On the 1st, 19th, and 33rd line, beginning of functions.
- On the 3rd to 5th, 23rd, 24th, 37th, and 38th line, declaration of local variables.
- On the 6th line, declaration of functions.
- On the 8th line, definition of the polynomial variable.
- On the 9th line, entering a 3-degree real rational-function.
- On the 11th line, generation of a logarithmically spaced vector.
- On the 12th and 13th line, calling the multiple-output functions `Bode()` and `Nyquist()`.
- On the 20th, 21st, 34th, and 35th line, declaration of arguments of functions.
- On the 26th and 40th line, definition of the imaginary unit.
- On the 27th and 41st line, evaluation of rational-function arrays.
- On the 28th and 29th line, array operations
- On the 30th and 42nd line, returning the list which contains two elements.

```
   Func void main()
   {
     Polynomial s;
     Rational G;
05:  Array w, ga, ph, re, im;
     List Bode(), Nyquist();

     s = $;                          /* Polynomial variable */
     G = (s^2 + 3*s + 4)/(s^3 + 5*s^2 + 6*s + 7);
10:                                  /* Transfer function   */
     w = logspace(0.01, 100.0);      /* Range of frequency  */

     {ga, ph} = Bode(G, w);          /* Gain and phase      */
     {re, im} = Nyquist(G, w);       /* Real and Imag. part */
15:
     print [[w][re][im]] >> "nyquist.mat"; /* Save data */
     print [[w][ga][ph]] >> "bode.mat";    /* Save data */
   }

20:Func List Bode(G, w)
     Rational G;
     Array w;
   {
     Complex j;
25:  Array data, ga, ph;

     j = (0,1);                      /* Imaginary Unit   */
     data = eval(G, j*w);            /* G(j*w)           */
     ga = 20.0 * log10(abs(data)); /* Gain in decibels */
30:  ph = arg(data) * 180.0 / PI;  /* Phase in degrees */
     return {ga, ph};                /* Gain and Phase   */
   }

   Func List Nyquist(G, w)
35:  Rational G;
     Array w;
   {
     Complex j;
     Array data;
40:
     j = (0,1);                      /* Imaginary unit   */
     data = eval(G, j*w);            /* G(j*w)           */
     return {Re(data), Im(data)};  /* Re- and im- part */
   }
```

List 4: Bode Nyquist Frequency Response

17.5 Simulation

17.5.1 State Variable Feedback Control (1)

The following program does the continuous-time and hybrid-time simulation of a linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

to which the state feedback control

$$u = \begin{bmatrix} -1 & -1 & -1 \end{bmatrix} x$$

is applied.

Hybrid-time simulation means the simulation of continuous-time plants with the digital controller. This list is the program of the simulation. The functions `Ode()` and `OdeHybrid()` are capable of integrating a broad range of non-linear system functions of both explicit and implicit nature. The function `Ode()` does the continuous-time simulation and the function `OdeHybrid()` does the hybrid simulation. The former function takes five arguments: the initial time (`0.0`), the terminal time (`5.0`), a vector initial condition (`x0`), the reference (`diff_eqs`) to the function which calculates the value of the derivative (`dx`) from the time `t` and the values of `x` and `u` at this time, the reference (`link_eqs`) to the function which calculates the value of the input (`u`) from the time `t` and the value of `x` at this time. The last argument (`link_eqs`) can be omitted in case of zero input. In addition to the five arguments, the function `OdeHybrid()` requires the sampling interval (`dt`) between which the constant input (`u`) is applied to the system.

```
Func void main()
{
  Real dt;
  Matrix x0;
  Array TC, XC, UC, TH, XH, UH;
  void diff_eqs(), link_eqs();

  x0 = [1 1 1]'; /* Initial state     */
  {TC,XC,UC} = Ode(0.0, 5.0, x0, diff_eqs, link_eqs);

  dt = 0.01;     /* Sampling interval */
  x0 = [1 1 1]'; /* Initial state     */
  {TH,XH,UH} =
     OdeHybrid(0.0, 5.0, dt, x0, diff_eqs,link_eqs);

  print [[TC][XC][UC]] >> "cont.mat";
  print [[TH][XH][UH]] >> "hybd.mat";
}

Func void diff_eqs(dx, t, x, u)
  Real t;
  Matrix x, dx, u;
{
  Matrix A, b;

  A = [[ 0   1   0]   /* System matrix   */
       [ 0   0   1]
       [-2, -3, -4]];
  b = [0 0 1]';       /* Input matrix    */
  dx = A * x + b * u; /* System equation */
}

Func void link_eqs(u, t, x)
  Real t;
  Matrix u, x;
{
  Matrix f;

  f = [-1, -1, -1];   /* State feedback control-law */
  u = f * x;
}
```


List 5: Simulation of State Variable Feedback Control (1)

17.5.2 State Variable Feedback Control (2)

The following program does the hybrid-time simulation of the linear system above by using 4th order Runge-kutta method directory.

```
Func void main()
{
  Integer i, ii;
  Real dt, t, t1, t2;
  Matrix x, u, controller();
  Array X, U;
  void plant();

  t1 = 0.0;  /* Initial time  */
  t2 = 5.0;  /* Terminal time */
  dt = 0.01; /* Time Interval */
  ii = Integer((t2 - t1) / dt);

  x = [1 1 1]';  /* Initial state */
  u = [0];

  X = Z(1, ii, x);
  U = Z(1, ii, u);

  t = t1;
  X(0, 0, x) = x;

  for (i = 1; i < ii; i++) {
    u = controller(t, x);
    x = rngkut4(t, x, plant, u, dt);

    U(0, i-1, u) = u;
    X(0, i, x) = x;
    t = t + dt;
  }
  u = controller(t, x);
  U(0, ii-1, u) = u;

  print [[X][U]] >> "cont.mat";
}
```



```
Func void plant(dx, t, x, u)
  Real t;
  Matrix x, dx, u;
{
  Matrix A, b;

  A = [[ 0   1   0]   /* System matrix   */
       [ 0   0   1]
       [-2, -3, -4]];

  b = [0 0 1]';       /* Input matrix    */

  dx = A * x + b * u; /* System equation */
}

Func Matrix controller(t, x)
  Real t;
  Matrix x;
{
  Matrix f, u;

  f = [-1, -1, -1];   /* State feedback control-law */
  u = f * x;
  return u;
}
```

List 6: Simulation of State Variable Feedback Control (2)

17.5.3 Simulation of Dynamic Feedback Control

The following program does the simulation of the closed-loop system with a strictly proper system and a proper controller.

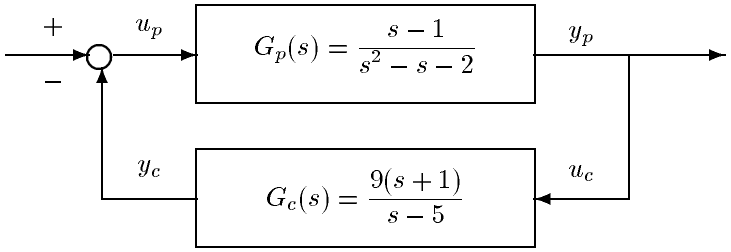

Figure 17.1: Plant and Controller


```
Matrix Ap, bp, cp;     /* Plant Parameters      */
Real   Ac, bc, cc, dc; /* Controller Parameters */

Func void init_params()
{
  /* Plant Parameters */
  Ap = [[0  1]
        [2  1]];
  bp = [0 1]';
  cp = [-1, 1];

  /* Controller Parameters */
  Ac = 5.0;
  bc = 1.0;
  cc = 54.0;
  dc = 9.0;
}

Func void main()
{
  Matrix x0;
  Array TC, XC, UC;

  void diff_eqs(), link_eqs(), init_params();

  init_params();

  x0 = [1 0 0]';
  {TC, XC, UC} =
     OdeAuto(0.0, 20.0, x0, diff_eqs, link_eqs);

  print [[TC][XC][UC]] >> "cont.mat";
}
```

```
Func void diff_eqs(DX, t, X, UY)
  Real t;
  Matrix X, DX, UY;
{
  Matrix xp, dxp;
  Real xc, dxc, up, uc;

  xp = X(1:2,1);          /* State of plant      */
  up = UY(1,1);           /* Input to plant      */
  xc = X(3,1);            /* State of controller */
  uc = UY(2,1);           /* Input to controller */

  dxp = Ap * xp + bp * up; /* System eq. of plant      */
  dxc = Ac * xc + bc * uc; /* System eq. of controller */

  DX = [[dxp][dxc]];
}

Func void link_eqs(UY, t, X)
  Real t;
  Matrix UY, X;
{
  Matrix xp;
  Real up, yp, xc, yc, uc;

  xp = X(1:2,1);          /* State of plant         */
  xc = X(3,1);            /* State of controller    */

  yp = Real(cp * xp);     /* Output from plant      */
  uc = yp;                /* Link 'yp' to 'uc'      */

  yc = cc * xc + dc * uc; /* Output from controller */
  up = - yc;              /* Link 'yc' to 'up'      */

  UY = [up uc]';
}
```

List 7: Simulation of Dynamic Feedback Control

17.5.4 Simulation of complex systems

We describe here how to write a program for simulation of complex systems. When we deal with the simulation of a complex system, it takes much time to write codes for extracting each state from an augumented state `X` at the top of the functions `diff_eqs()` and `link_eqs()`. Two functions `VectorConnect()` and `VectorChop()` are useful for it and are introduced in the following example program.

Each block P, D, C, and F are a plant, an uncertainty of the plant, a feedforward controller, and a feedback controller. They are described by a state equation and a output equation, where each coefficient of the equations are (Ap, Bp, Cp), (Ad, Bd, Cd), (Ac, Bc, Cc), and (Af, Bf, Cf).

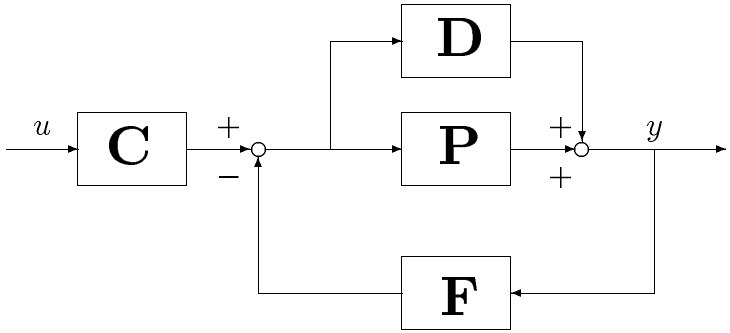

Figure 17.2: Simulation of a complex system

```
Matrix Ap, Bp, Cp; /* Plant Parameters          */
Matrix Ad, Bd, Cd; /* Uncertainty Parameters    */
Matrix Ac, Bc, Cc; /* Controller-1 Parameters   */
Matrix Af, Bf, Cf; /* Controller-2 Parameters   */
List state_idx;    /* List of indices of states */
List input_idx;    /* List of indices of inputs */

Func void main()
{
  Matrix xp0, xd0, xc0, xf0, x0;
  Matrix up0, ud0, uc0, uf0, u0;
  Array TC, XC, UC;

  void diff_eqs(), link_eqs();

  read Ap, Bp, Cp <- "plant";
  read Ad, Bd, Cd <- "uncertainty";
  read Ac, Bc, Cc <- "controller-1";
  read Af, Bf, Cf <- "controller-2";
  read xp0, xd0, xc0, xf0 <- "init-states";
  read up0, ud0, uc0, uf0 <- "init-inputs";

  {x0, state_idx} = VectorConnect({xp0, xd0, xc0, xf0});
  {x0, input_idx} = VectorConnect({up0, ud0, uc0, uf0});
  {TC, XC, UC} =
     Ode45Auto(0.0, 20.0, x0, diff_eqs, link_eqs);

  print [[TC][XC][UC]] >> "cont.mat";
}
```



```
Func void diff_eqs(DX, t, X, U)
  Real t;
  Matrix X, DX, U;
{
  Matrix xp, xd, xc, xf;
  Matrix dxp, dxd, dxc, dxf;
  Matrix up, ud, uc, uf;

  {xp, xd, xc, xf} = VectorChop(X, state_idx);
  {up, ud, uc, uf} = VectorChop(U, input_idx);

  dxp = Ap * xp + Bp * up;
  dxd = Ad * xd + Bd * ud;
  dxc = Ac * xc + Bc * uc;
  dxf = Af * xf + Bf * uf;
  DX = [[dxp][dxd][dxc][dxf]];
}

Func void link_eqs(U, t, X)
  Real t;
  Matrix U, X;
{
  Matrix xp, xd, xc, xf;
  Matrix up, ud, uc, uf;
  Matrix yp, yd, yc, yf;

  {xp, xd, xc, xf} = VectorChop(X, state_idx);

  yp = Ap * xp;
  yd = Ad * xd;
  yc = Ac * xc;
  yf = Af * xf;

  up = yc - yf;
  ud = up;
  uc = u;
  uf = yp + yd;
  U = [[up][ud][uc][uf]]';
}
```

List 8: Simulation of a complex system

Bibliography

[1] George E. Forsythe, Michael A. Malcolm, and Cleve B. Moler. *Computer Methods for Mathematical Computations.* Prentice-Hall, 1977.

[2] B.S. Garbow, J.M. Boyle, J.J. Dongarra, and C.B. Moler. *Matrix Eigensystem Routines – EISPACK Guide Extension.* Springer-Verlag, 1977.

[3] Gene H. Golub and Charles F. Van Loan. *Matrix Computations.* The Johns Hopkins University Press, 1989.

[4] J.E.Potter. Matrix quadratic solution. *SIAM Journal on Appl. Math.*, 14:496–501, 1966.

[5] Brian W. Kernighan and Dennis M. Ritchie. *The C Programming Language.* Prentice-Hall, 1978.

[6] Masanobu Koga and Katsuhisa Furuta. MaTX : A high-performance interactive software package for scientific and engineering computation. *Proc. of CADCS'91, Swansea, U.K.*, pages 39–44, 1991.

[7] Masanobu Koga and Katsuhisa Furuta. MaTX : A high-performance programming language (interpreter and compiler) for scientific and engineering computation. *Proc. of CACSD'92, Napa, U.S.A.*, pages 15–22, 1992.

[8] C. Moler, J. Littel, and S. Bangert. *PC-MATLAB – User's Guide.* N. Main St., Sherborn, MA 01770, USA, 3.1 edition, 1987.

[9] Cleve Moler. *MATLAB – User's Guide.* Alberquerque, USA, 1980.

[10] William H. Press, Brian P. Flannery, Saul A. Teukolsky, and William T. Veterling. *Numerical Recipes in C (The Art of Scientific Computing).* Cambridge University Press, 1988.

[11] B.T. Smith, J.M. Boyle, J.J. Dongarra, B.S. Garbow, Y. Ikebe, V.C. Klema, and C.B. Moler. *Matrix Eigensystem Routines – EISPACK Guide.* Springer-Verlag, 1976.

[12] G.W. Stewart. *Introduction To Matrix Computations.* Academic Press, 1973.

[13] 近藤 嘉雪. *yacc* による *C* コンパイラプログラミング. ソフトバンク, 1990.

[14] 石田 晴久 監訳. プログラミング言語 *C*. 共立出版株式会社, 1981.

[15] 石田 晴久 監訳. *UNIX* プログラミング環境. アスキー出版局, 1985.

Appendix A

Data Format

A.1 MAT Data Format

1. The first line begins with a character "`#`", and two integers, i.e. the number of rows and the number of columns, follow.
2. For a complex matrix, the capital "`C`" follows next.
3. Data is separated by blanks, tabs, or newlines.

The MAT data format of a real matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

is

```
# 3 4
  1.0000   5.0000   9.0000
  2.0000   6.0000   10.000
  3.0000   7.0000   11.000
  4.0000   8.0000   12.000
```


and the MAT data format of a complex matrix

$$\begin{bmatrix} 1+2i & 3+4i & 5+6i \\ 7+8i & 9+10i & 11+12i \end{bmatrix}$$

is

```
# 2 3 C
  1.0000   2.0000   7.0000   8.0000
  3.0000   4.0000   9.0000   10.000
  5.0000   6.0000   11.000   12.000
```

A.2 MX Data Format

Datafile contains more than one variable, which are stored successively. Each data starts with a 20-byte header which contains the properties of the variables. The 20-byte header consists of five 4-bytes integers.

1. Integer which indicates the type of the variable. Let represent this integer as `[M|O|P|C]`.

   "`M`" denotes the type of machine where this data was stored.

Machine	M
Sun	0
Appolo	0
Macintosh	0
PC	1
Other Intel	1
VAX D-float	2
VAX G-float	3

   "`O`" indicates the order of matrix data stored. "`O`" is 0, if column first. "`O`" is 1, if row first.

   "`P`" indicates the precision of the data.

Data Precision	P
Double-precision real number	0
Single-precision real number	1
Signed 32-bit integer	2
Signed 16-bit integer	3
Unsigned 16-bit integer	4

"C" indicates the class of the data.

Data Class	C
Matrix	0
Integer	1
Real Number	2
Complex Number	3
String	4
Polynomial	5
Rational	6
List	7
Array	8
Index	9

2. The second integer depends on the class of data.

Data Class	Name	Content
Matrix	rows	Row number
Integer	dummy	
Real Number	dummy	
Complex Number	dummy	
String	stringlen	Length of string
Polynomial	degree	Degree
Rational	ndegree	Degree of Numerator
List	length	Length of element
Array	rows	Row number
Index	rows	Row number

3. The third integer depends on the class of data.

Data Class	Name	Content
Matrix	cols	Column number
Integer	dummy	
Real Number	dummy	
Complex Number	dummy	
String	dummy	
Polynomial	dummy	
Rational	ddegree	Degree of Denominator
List	dummy	
Array	cols	Column number
Index	cols	Column number

4. The fourth integer depends on the class of data.

Data Class	Name	Content
Matrix	class	Class of matrix
Integer	dummy	
Real Number	dummy	
Complex Number	dummy	
String	dummy	
Polynomial	imagf	Real or Complex
Rational	imagf	Real or Complex
List	dummy	
Array	class	Class of matrix
Index	class	REAL NUMBER

5. The fifth integer indicates the length of the name of variable.

Data Class	Name	Content
Matrix	namelen	Length of name
Integer	namelen	Length of name
Real Number	namelen	Length of name
Complex Number	namelen	Length of name
String	namelen	Length of name
Polynomial	namelen	Length of name
Rational	namelen	Length of name
List	namelen	Length of name
Array	namelen	Length of name
Index	namelen	Length of name

The name of variable, whose length is `namelen`, follows the 20-byte header, and the last character is `NUL`.
The following data depends on the class of variable.

Integer Integer (2 bytes or 4 bytes).

Real Number Real number (4 bytes or 8 bytes).

Complex Number Two real numbers (4 bytes or 8 bytes).

String `stringlen`-length string. The last character is `NUL`.

Polynomial

- **Real Polynomial** `degree + 1` real numbers (4 bytes or 8 bytes).
- **Complex Polynomial** `2×(degree + 1)` real numbers (4 bytes or 8 bytes).

Rational Function

- **Real Rational Function** `ndegree + 1` real numbers and `ddegree + 1` real numbers (4 bytes or 8 bytes).
- **Complex Rational Function** `2×(ndegree + 1)` real numbers and `2×(ddegree + 1)` real numbers (4 bytes or 8 bytes).

Matrix

- **Real Matrix** `rows×cols` real numbers (4 bytes or 8 bytes).
- **Complex Matrix** `2×(rows×cols)` real numbers (4 bytes or 8 bytes).
- **Real Polynomial Matrix** `rows×cols` real polynomials: `degree + 1` real numbers (4 bytes or 8 bytes).

- **Complex Polynomial Matrix** `rows×cols` complex polynomials: `degree + 1` real numbers (4 bytes or 8 bytes).
- **Real Rational Function Matrix** `rows×cols` real rational functions: `ndegree + 1` real numbers and `ddegree + 1` real numbers. (4 bytes or 8 bytes).
- **Complex Rational Function Matrix** `rows×cols` complex rational functions: `2×(ndegree + 1)` real numbers and `2×(ddegree + 1)` real numbers (4 bytes or 8 bytes).

List `length` 4-byte integer which indicates the classed of the elements, and the combination of the elements.

Array

- **Real Array** `rows×cols` real numbers (4 bytes or 8 bytes).
- **Complex Array** `2×(rows×cols)` real numbers (4 bytes or 8 bytes).
- **Real Polynomial Array** `rows×cols` real polynomials: `degree + 1` real numbers (4 bytes or 8 bytes).
- **Complex Polynomial Array** `rows×cols` complex polynomials: `degree + 1` real numbers (4 bytes or 8 bytes).
- **Real Rational Function Array** `rows×cols` real rational functions: `ndegree + 1` real numbers and `ddegree + 1` real numbers. (4 bytes or 8 bytes).

Complex Rational Function Array	`rows`×`cols` complex rational functions: `2`×`(ndegree + 1)` real numbers and `2`×`(ddegree + 1)` real numbers (4 bytes or 8 bytes).
Index	`cols` real numbers (4 bytes or 8 bytes).

Appendix B

Operation

The binary arithmetic operators are +, -, *, .*, /, ./, \, .\, ^, .^. The + and - operators have the same precedence, which is lower than the (identical) precedence of *, .*, /, ./, \, .\, ^, and .^, which are in turn lower than that of unary minus -, inverse ~, transpose', and conjugate transpose #. Arithmetic operators group left to right. Table 1 below summarizes the rules for precedence and associativity of all operators.

Table B.1: Precedence and Associativity of Operators

Operator	Associativity
( ) $ [ ]	left to right
++ --	right to left
^ ~ ' # .^ .~	right to left
@ ! -	right to left
* / \ .* ./ .\	left to right
+ -	left to right
< <= > >=	left to right
== !=	left to right
&&	left to right
||	left to right
=	right to left

Operators on the same line have the same precedence; rows are in order

of decreasing precedence, so, for example, * and / have the same precedence which is higher than that of + and -.

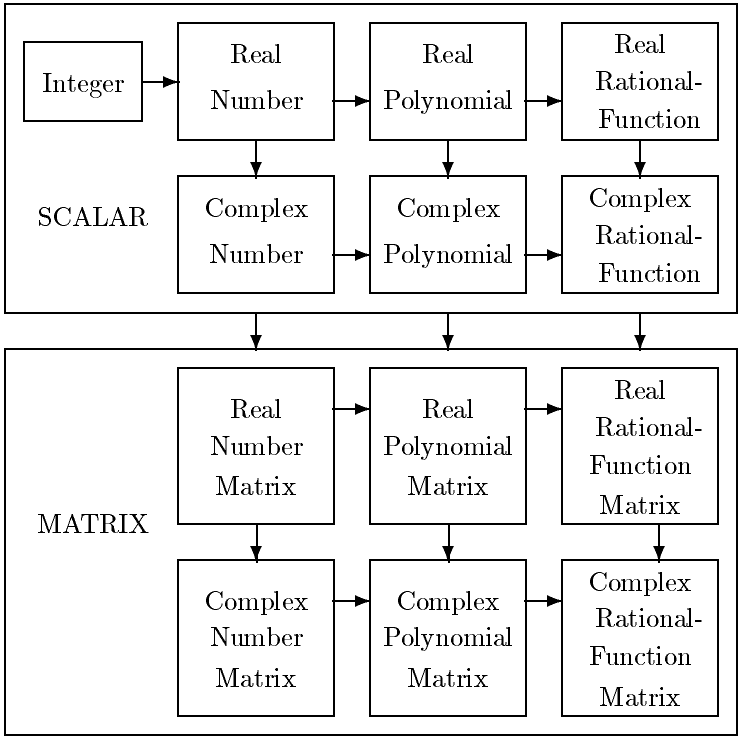

Figure B.1: The Automatic Class Conversion Rule

In case of operations between distinct class objects, "Class Conversion" is done according to the "automatic Class Conversion Rule" below. The class of the operation result, except scalar multiplication and power, is the same as that of the higher class one. Arithmetic operations of arrays are element-by-element in stead of the usual algebraic matrix operations, and the result is a new array whose elements are the result of element-by-element operation.

B.1 Addition

```
Integer                 = Integer + Integer
Real Number             = Real Number + Real Number
Complex Number          = Complex Number + Complex Number
Real-Poly               = Real-Poly + Real-Poly
Comp-Poly               = Comp-Poly + Comp-Poly
Real-Rat-Func           = Real-Rat-Func + Real-Rat-Func
Comp-Rat-Func           = Comp-Rat-Func + Comp-Rat-Func
Real(Mat|Arr)           = Real(Mat|Arr) + Real(Mat|Arr)
Complex(Mat|Arr)        = Complex(Mat|Arr) + Complex(Mat|Arr)
Real-Poly(Mat|Arr)      = Real-Poly(Mat|Arr) + Real-Poly(Mat|Arr)
Comp-Poly(Mat|Arr)      = Comp-Poly(Mat|Arr) + Comp-Poly(Mat|Arr)
Real-Rat-Func(Mat|Arr)  = Real-Rat-Func(Mat|Arr)
                                    + Real-Rat-Func(Mat|Arr)
Comp-Rat-Func(Mat|Arr)  = Comp-Rat-Func(Mat|Arr)
                                    + Comp-Rat-Func(Mat|Arr)
String                  = String + String
```

B.2 Subtraction

```
Integer                 = Integer - Integer
Real Number             = Real Number - Real Number
Complex Number          = Complex Number - Complex Number
Real-Poly               = Real-Poly - Real-Poly
Comp-Poly               = Comp-Poly - Comp-Poly
Real-Rat-Func           = Real-Rat-Func - Real-Rat-Func
Comp-Rat-Func           = Comp-Rat-Func - Comp-Rat-Func
Real(Mat|Arr)           = Real(Mat|Arr) - Real(Mat|Arr)
Complex(Mat|Arr)        = Complex(Mat|Arr) - Complex(Mat|Arr)
Real-Poly(Mat|Arr)      = Real-Poly(Mat|Arr) - Real-Poly(Mat|Arr)
Comp-Poly(Mat|Arr)      = Comp-Poly(Mat|Arr) - Comp-Poly(Mat|Arr)
Real-Rat-Func(Mat|Arr)  = Real-Rat-Func(Mat|Arr)
                                    - Real-Rat-Func(Mat|Arr)
Comp-Rat-Func(Mat|Arr)  = Comp-Rat-Func(Mat|Arr)
                                    - Comp-Rat-Func(Mat|Arr)
```


B.3 Multiplication

Even if the multiplication result of matrices becomes a `1 x 1` matrix, it is not a scalar value but still a matrix.

```
Integer                 = Integer * Integer
Real Number             = Real Number * Real Number
Complex Number          = Complex Number * Complex Number
Real-Poly               = Real-Poly * Real-Poly
Comp-Poly               = Comp-Poly * Comp-Poly
Real-Rat-Func           = Real-Rat-Func * Real-Rat-Func
Comp-Rat-Func           = Comp-Rat-Func * Comp-Rat-Func
Real(Mat|Arr)           = Real(Mat|Arr) * Real(Mat|Arr)
Complex(Mat|Arr)        = Complex(Mat|Arr) * Complex(Mat|Arr)
Real-Poly(Mat|Arr)      = Real-Poly(Mat|Arr) * Real-Poly(Mat|Arr)
Comp-Poly(Mat|Arr)      = Comp-Poly(Mat|Arr) * Comp-Poly(Mat|Arr)
Real-Rat-Func(Mat|Arr)  = Real-Rat-Func(Mat|Arr)
                                    * Real-Rat-Func(Mat|Arr)
Comp-Rat-Func(Mat|Arr)  = Comp-Rat-Func(Mat|Arr)
                                    * Comp-Rat-Func(Mat|Arr)
Real Matrix             = Real Matrix .* Real(Mat|Arr)
Complex Matrix          = Complex Matrix .* Complex Matrix
Real-PolyMatrix         = Real-PolyMatrix .* Real-PolyMatrix
Comp-PolyMatrix         = Comp-PolyMatrix .* Comp-PolyMatrix
Real-Rat-FuncMatrix     = Real-Rat-FuncMatrix .* Real-Rat-FuncMatrix
Comp-Rat-FuncMatrix     = Comp-Rat-FuncMatrix .* Comp-Rat-FuncMatrix
```

B.4 Scalar Multiplication

```
Real(Mat|Arr)           = (Integer|Real Number) * Real(Mat|Arr)
Real(Mat|Arr)           = Real(Mat|Arr) * (Integer|Real Number)
Complex(Mat|Arr)        = (Integer|Real Number) * Complex(Mat|Arr)
Complex(Mat|Arr)        = Complex(Mat|Arr) * (Integer|Real Number)
Complex(Mat|Arr)        = Complex Number * Real(Mat|Arr)
Complex(Mat|Arr)        = Real(Mat|Arr) * Complex Number
Complex(Mat|Arr)        = Complex Number * Complex(Mat|Arr)
Complex(Mat|Arr)        = Complex(Mat|Arr) * Complex Number
```

```
Real-Poly(Mat|Arr)     = Real-Poly * Real(Mat|Arr)
Real-Poly(Mat|Arr)     = Real(Mat|Arr) * Real-Poly
Comp-Poly(Mat|Arr)     = Real-Poly * Complex(Mat|Arr)
Comp-Poly(Mat|Arr)     = Complex(Mat|Arr) * Real-Poly
Comp-Poly(Mat|Arr)     = Comp-Poly * Real(Mat|Arr)
Comp-Poly(Mat|Arr)     = Real(Mat|Arr) * Comp-Poly
Comp-Poly(Mat|Arr)     = Comp-Poly * Complex(Mat|Arr)
Comp-Poly(Mat|Arr)     = Complex(Mat|Arr) * Comp-Poly


Real-Rat-Func(Mat|Arr) = Real-Rat-Func * Real(Mat|Arr)
Real-Rat-Func(Mat|Arr) = Real(Mat|Arr) * Real-Rat-Func
Comp-Rat-Func(Mat|Arr) = Real-Rat-Func * Complex(Mat|Arr)
Comp-Rat-Func(Mat|Arr) = Complex(Mat|Arr) * Real-Rat-Func
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func * Real(Mat|Arr)
Comp-Rat-Func(Mat|Arr) = Real(Mat|Arr) * Comp-Rat-Func
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func * Complex(Mat|Arr)
Comp-Rat-Func(Mat|Arr) = Complex(Mat|Arr) * Comp-Rat-Func


Real-Poly(Mat|Arr)     = (Integer|Real Number) * Real-Poly(Mat|Arr)
Real-Poly(Mat|Arr)     = Real-Poly(Mat|Arr) * (Integer|Real Number)
Comp-Poly(Mat|Arr)     = (Integer|Real Number) * Comp-Poly(Mat|Arr)
Comp-Poly(Mat|Arr)     = Comp-Poly(Mat|Arr) * (Integer|Real Number)
Comp-Poly(Mat|Arr)     = Complex Number * Real-Poly(Mat|Arr)
Comp-Poly(Mat|Arr)     = Real-Poly(Mat|Arr) * Complex Number
Comp-Poly(Mat|Arr)     = Complex Number * Comp-Poly(Mat|Arr)
Comp-Poly(Mat|Arr)     = Comp-Poly(Mat|Arr) * Complex Number


Real-Rat-Func(Mat|Arr) = (Integer|Real Number)
                                 * Real-Rat-Func(Mat|Arr)
Real-Rat-Func(Mat|Arr) = Real-Rat-Func(Mat|Arr)
                                 * (Integer|Real Number)
Comp-Rat-Func(Mat|Arr) = (Integer|Real Number)
                                 * Comp-Rat-Func(Mat|Arr)
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func(Mat|Arr)
                                 * (Integer|Real Number)
Comp-Rat-Func(Mat|Arr) = Complex Number * Real-Rat-Func(Mat|Arr)
Comp-Rat-Func(Mat|Arr) = Real-Rat-Func(Mat|Arr) * Complex Number
Comp-Rat-Func(Mat|Arr) = Complex Number * Comp-Rat-Func(Mat|Arr)
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func(Mat|Arr) * Complex Number
```



```
String = Integer * String
String = String * Integer
```

B.5 Inverse

```
Real Number      = Real Number    ~ = 1.0 / Real Number
Complex Number   = Complex Number ~ = 1.0 / Complex Number
Real Matrix      = Real Matrix ~
Complex Matrix   = Complex Matrix ~
Real Array       = Real Array ~
Complex Array    = Complex Array ~
Rat-FuncArray    = PolyArray ~
Rat-FuncArray    = Rat-FuncArray ~
Real-Rat-Func    = Real-Poly ~
Comp-Rat-Func    = Comp-Poly ~
Real-Rat-Func    = Real-Rat-Func ~
Comp-Rat-Func    = Comp-Rat-Func ~
String           = String ~

Real Matrix      = Real Matrix .~
Complex Matrix   = Complex Matrix .~
Rat-Func Matrix  = Poly-Matrix .~
Rat-Func Matrix  = Rat-Func Matrix .~
Real Array       = Real Array .~
Complex Array    = Complex Array .~
Rat-FuncArray    = PolyArray .~
Rat-FuncArray    = Rat-FuncArray .~
```

B.6 Division

Integer division truncates any fractional part. In the division of two matrices, "`X = A \ B`" is a solution of the equation "`A * X = B`", and "`X = A / B`" is a solution of the equation "`X * B = A`". If `A` is a square matrix, the equation is solved by using LU decomposition, and otherwise the least square solution is calculated.

```
OP3 = OP1 / OP2 ----> (OP3 = OP1 * OP2~)
OP3 = OP1 \ OP2 ----> (OP3 = OP1~ * OP2)

Integer        = Integer / Integer
Real Number    = Real Number / Real Number
Complex Number = Complex Number / Complex Number

Real-Poly      = Real-Poly / (Integer|Real Number)
Comp-Poly      = Real-Poly / Complex Number
Comp-Poly      = Comp-Poly / (Integer|Real Number)
Comp-Poly      = Comp-Poly / Complex Number

Real-Rat-Func  = Real-Poly / Real-Poly
Comp-Rat-Func  = Comp-Poly / Comp-Poly
Real-Rat-Func  = Real-Rat-Func / Real-Rat-Func
Comp-Rat-Func  = Comp-Rat-Func / Comp-Rat-Func

Real(Mat|Arr)          = Real(Mat|Arr) / Real(Mat|Arr)
Complex(Mat|Arr)       = Complex(Mat|Arr) / Complex(Mat|Arr)
Real-Poly(Mat|Arr)     = Real-Poly(Mat|Arr) / (Integer|Real Number)
Comp-Poly(Mat|Arr)     = Real-Poly(Mat|Arr) / Complex Number
Comp-Poly(Mat|Arr)     = Comp-Poly(Mat|Arr) / (Integer|Real Number)
Comp-Poly(Mat|Arr)     = Comp-Poly(Mat|Arr) / Complex Number
Real-Rat-Func(Mat|Arr) = Real-Rat-Func(Mat|Arr) / (Int|Real Number)
Comp-Rat-Func(Mat|Arr) = Real-Rat-Func(Mat|Arr) / Complex Number
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func(Mat|Arr) / (Int|Real Number)
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func(Mat|Arr) / Complex Number

Real Matrix       = Real Matrix ./ Real Matrix
Complex Matrix    = Complex Matrix ./ Complex Matrix
Rat-Func Matrix   = (Real|Comp|Poly|Rat-Func)Matrix ./ Poly-Matrix

Integer           = Integer \ Integer
Real Number       = Real Number \ Real Number
Complex Number    = Complex Number \ Complex Number

Real-Rat-Func     = Real-Poly \ (Integer|Real Number)
Comp-Rat-Func     = Real-Poly \ Complex Number
Comp-Rat-Func     = Comp-Poly \ (Integer|Real Number)
Comp-Rat-Func     = Comp-Poly \ Complex Number
```



```
Real-Rat-Func     = Real-Poly \ Real-Poly
Comp-Rat-Func     = Comp-Poly \ Comp-Poly
Real-Rat-Func     = Real-Rat-Func \ Real-Rat-Func
Comp-Rat-Func     = Comp-Rat-Func \ Comp-Rat-Func

Real(Mat|Arr)     = Real(Mat|Arr) \ Real(Mat|Arr)
Complex(Mat|Arr)  = Complex(Mat|Arr) \ Complex(Mat|Arr)

Real Matrix       = Real Matrix .\ Real Matrix
Complex Matrix    = Complex Matrix .\ Complex Matrix
Rat-Func Matrix   = Poly-Matrix .\ (Real|Comp|Poly|Rat-Func)Matrix
```

B.7 Power

When the index is negative, the operation means its reciprocal (inverse) to the absolute of the index. It should be noted that the inverse of a polynomial is a rational function.

```
Integer                = Integer ^ Integer
Real Number            = Real Number ^ Real Number
Complex Number         = Complex Number ^ Complex Number
Real-Poly              = Real-Poly ^ Integer
Comp-Poly              = Comp-Poly ^ Integer
Real-Rat-Func          = Real-Rat-Func ^ Integer
Comp-Rat-Func          = Comp-Rat-Func ^ Integer
Real(Mat|Arr)          = Real(Mat|Arr) ^ Integer
Complex(Mat|Arr)       = Complex(Mat|Arr) ^ Integer
Real-Poly(Mat|Arr)     = Real-Poly(Mat|Arr) ^ Integer
Comp-Poly(Mat|Arr)     = Comp-Poly(Mat|Arr) ^ Integer
Real-Rat-Func(Mat|Arr) = Real-Rat-Func(Mat|Arr) ^ Integer
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func(Mat|Arr) ^ Integer


Real Array             = Real Array ^ Real
Complex Array          = Real Array ^ Complex
Complex Array          = Complex Array ^ Real
Complex Array          = Complex Array ^ Complex


Real Matrix            = Real Array ^ Real Array
```

```
Complex Matrix         = Complex Array ^ Real Array
Real-PolyMatrix        = Real-PolyArray ^ Real Array
Comp-PolyMatrix        = Comp-PolyArray ^ Real Array
Real-Rat-FuncMatrix    = Real-Rat-FuncArray ^ Real Array
Comp-Rat-FuncMatrix    = Comp-Rat-FuncArray ^ Real Array


Real Matrix            = Real Matrix .^ Integer
Complex Matrix         = Complex Matrix .^ Integer
Real-PolyMatrix        = Real-PolyMatrix .^ Integer
Comp-PolyMatrix        = Comp-PolyMatrix .^ Integer
Real-Rat-FuncMatrix    = Real-Rat-FuncMatrix .^ Integer
Comp-Rat-FuncMatrix    = Comp-Rat-FuncMatrix .^ Integer


Real Matrix            = Real Matrix .^ Real
Complex Matrix         = Complex Matrix .^ Real
Complex Matrix         = Complex Matrix .^ Complex


Real Matrix            = Real Matrix .^ Real Matrix
Complex Matrix         = Complex Matrix .^ Real Matrix
Real-PolyMatrix        = Real-PolyMatrix .^ Real Matrix
Comp-PolyMatrix        = Comp-PolyMatrix .^ Real Matrix
Real-Rat-FuncMatrix    = Real-Rat-FuncMatrix .^ Real Matrix
Comp-Rat-FuncMatrix    = Comp-Rat-FuncMatrix .^ Real Matrix


Real Matrix            = (Integer|Real Number) .^ Real Matrix
Complex Matrix         = Complex Number .^ Real Matrix
Real-PolyMatrix        = Real-Poly .^ Real Matrix
Comp-PolyMatrix        = Comp-Poly .^ Real Matrix
Real-Rat-FuncMatrix    = Real-Rat-Func .^ Real Matrix
Comp-Rat-FuncMatrix    = Comp-Rat-Func .^ Real Matrix
```

B.8 Transpose

Transpose of scalar values (Integer, Real Number, Polynomial, Rational Function) is the same as itself.


```
Integer                = Integer'
Real Number            = Real Number'
Complex Number         = Complex Number'
Real-Poly              = Real-Poly'
Comp-Poly              = Comp-Poly'
Real-Rat-Func          = Real-Rat-Func'
Comp-Rat-Func          = Comp-Rat-Func'
Real(Mat|Arr)          = Real(Mat|Arr)'
Complex(Mat|Arr)       = Complex(Mat|Arr)'
Real-Poly(Mat|Arr)     = Real-Poly(Mat|Arr)'
Comp-Poly(Mat|Arr)     = Comp-Poly(Mat|Arr)'
Real-Rat-Func(Mat|Arr) = Real-Rat-Func(Mat|Arr)'
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func(Mat|Arr)'
```

B.9 Complex Conjugate Transpose

Complex conjugate transpose of Integer, Real Number, and Real (Polynomial Rational Function) is the same as itself. Real(Matrix and Array) mean the transpose of themselves.

```
Integer                = Integer' = Integer#
Real Number            = Real Number' = Real Number#
Complex Number         = Complex Number#
Real-Poly              = Real-Poly'= Real-Poly#
Comp-Poly              = Comp-Poly#
Real-Rat-Func          = Real-Rat-Func'= Real-Rat-Func#
Comp-Rat-Func          = Comp-Rat-Func#
Real(Mat|Arr)          = Real(Mat|Arr)' = Real(Mat|Arr)#
Complex(Mat|Arr)       = Complex(Mat|Arr)#
Real-Poly(Mat|Arr)     = Real-Poly(Mat|Arr)' = Real-Poly(Mat|Arr)#
Comp-Poly(Mat|Arr)     = Comp-Poly(Mat|Arr)#
Real-Rat-Func(Mat|Arr) = Real-Rat-Func(Mat|Arr)'
                       = Real-Rat-Func(Mat|Arr)#
Comp-Rat-Func(Mat|Arr) = Comp-Rat-Func(Mat|Arr)#
```

B.10 Negation

```
Integer                = - Integer
Real Number            = - Real Number
Complex Number         = - Complex Number
Real-Poly              = - Real-Poly
Comp-Poly              = - Comp-Poly
Real-Rat-Func          = - Real-Rat-Func
Comp-Rat-Func          = - Comp-Rat-Func
Real(Mat|Arr)          = - Real(Mat|Arr)
Complex(Mat|Arr)       = - Complex(Mat|Arr)
Real-Poly(Mat|Arr)     = - Real-Poly(Mat|Arr)
Comp-Poly(Mat|Arr)     = - Comp-Poly(Mat|Arr)
Real-Rat-Func(Mat|Arr) = - Real-Rat-Func(Mat|Arr)
Comp-Rat-Func(Mat|Arr) = - Comp-Rat-Func(Mat|Arr)
```

B.11 Increment and Decrement Operators

MATX provides two operators for incrementing and decrementing variables. The increment operator "++" adds 1 to its operand; the decrement operator "--" subtracts 1. The interesting aspect is that "++" and "--" may be used either as prefix operators (before the variable, as in "++n)", or postfix (after the variable "n++)". In both cases, the effect is to increment n.

```
Integer++         /* Integer = Integer + 1           */
Real Number++     /* Real Number = Real Number + 1.0 */
Integer--         /* Integer = Integer - 1           */
Real Number--     /* Real Number = Real Number - 1.0 */
++Integer         /* Integer = Integer + 1           */
++Real Number     /* Real Number = Real Number + 1.0 */
--Integer         /* Integer = Integer - 1           */
--Real Number     /* Real Number = Real Number - 1.0 */
```

B.12 Relational Operation

The relational operators are >, >=, <, <=. They all have the same precedence. Just below them in precedence are the equality operators ==, !=.

which have the same precedence. Relationals have lower precedence than arithmetic operators.

`Integer1 < Integer2`	`Integer1 is less than Integer2`
`Integer1 > Integer2`	`Integer1 is bigger than Integer2`
`Integer1 <= Integer2`	`Integer1 is less than or equal to Integer2`
`Integer1 >= Integer2`	`Integer1 is bigger or equal to Integer2`
`Integer1 == Integer2`	`Integer1 is equal to Integer2`
`Integer1 != Integer2`	`Integer1 is not equal to Integer2`
`Real-Num1 < Real-Num2`	`Real-Num1 is less than Real-Num2`
`Real-Num1 > Real-Num2`	`Real-Num1 is bigger than Real-Num2`
`Real-Num1 <= Real-Num2`	`Number1 is less than or equal to Number2`
`Real-Num1 >= Real-Num2`	`Number1 is bigger or equal to Number2`
`Real-Num1 == Real-Num2`	`Real-Num1 is equal to Real-Num2`
`Real-Num1 != Real-Num2`	`Num1 is not equal to Num2`
`Matrix < 0`	`maxeigval(Matrix) < 0`
`Matrix <= 0`	`maxeigval(Matrix) <= 0`
`Matrix > 0`	`mineigval(Matrix) > 0`
`Matrix >= 0`	`mineigval(Matrix) >= 0`
`Matrix == 0`	`Matrix is a Zero Matrix`
`Matrix != 0`	`Matrix is not a Zero Matrix`
`0 < Matrix`	`mineigval(Matrix) > 0`
`0 <= Matrix`	`mineigval(Matrix) >= 0`
`0 > Matrix`	`maxeigval(Matrix) < 0`
`0 >= Matrix`	`maxeigval(Matrix) <= 0`
`0 == Matrix`	`Matrix is a Zero Matrix`
`0 != Matrix`	`Matrix is not a Zero Matrix`
`Matrix1 < Matrix2`	`maxeigval(Matrix1 - Matrix2) < 0`
`Matrix1 <= Matrix2`	`maxeigval(Matrix1 - Matrix2) <= 0`
`Matrix1 > Matrix2`	`mineigval(Matrix1 - Matrix2) > 0`
`Matrix1 >= Matrix2`	`mineigval(Matrix1 - Matrix2) >= 0`
`Matrix1 == Matrix2`	`Matrix1 is equal to Matrix2`
`Matrix1 != Matrix2`	`Matrix1 is not equal to Matrix2`

```
Array1 < Array2           Element-by-element relation:
Array1 > Array2           Element-by-element relation:
                          (0|1)-Array
Array1 <= Array2          Element-by-element relation:
                          (0|1)-Array
Array1 >= Array2          Element-by-element relation:
                          (0|1)-Array
Array1 == Array2          Element-by-element relation:
                          (0|1)-Array
Array1 != Array2          Element-by-element relation:
                          (0|1)-Array
(Int|Real-Num)< Array     Element-by-element relation:
(Int|Real-Num)> Array     Element-by-element relation:
                          (0|1)-Array
(Int|Real-Num)<= Array    Element-by-element relation:
                          (0|1)-Array
(Int|Real-Num)>= Array    Element-by-element relation:
                          (0|1)-Array
(Int|Real-Num)== Array    Element-by-element relation:
                          (0|1)-Array
(Int|Real-Num)!= Array    Element-by-element relation:
                          (0|1)-Array
Array1 < (Int|Real-Num)   Element-by-element relation:
Array1 > (Int|Real-Num)   Element-by-element relation:
                          (0|1)-Array
Array1 <= (Int|Real-Num)  Element-by-element relation:
                          (0|1)-Array
Array1 >= (Int|Real-Num)  Element-by-element relation:
                          (0|1)-Array
Array1 == (Int|Real-Num)  Element-by-element relation:
                          (0|1)-Array
Array1 != (Int|Real-Num)  Element-by-element relation:
                          (0|1)-Array
String1 == String2        String1 is same as String2
String1 != String2        String1 is not same as String2
Poly1 == Poly2            Poly1 is equal to Poly2
Poly1 != Poly2            Poly1 is not equal to Poly2
```



```
Rat-Func1 == Rat-Func2    Rat-Func1 is equal to Rat-Func2
Rat-Func1 != Rat-Func2    Func1 is not equal to Func2
```

B.13 Logical Operation

The precedence of `&&` is greater than that of `||`, and both are lower than relational and equality operators. Expressions connected by `&&` or `||` are evaluated left to right, and evaluation does not stop even if the truth or falsehood of the result is known. The unary negation operator `!` converts a non-zero or true operand into 0, and z zero or false operand into 1.

```
Integer1 && Integer2  (1 if both Int1 and Int2 are 1)
Ineger1 || Integer2   (1 if either Int1 or Int2 are 1)
!  Integer1           (1 if Int1 is 0)
Real1 && Real2        (1 if neither Real1 nor Real2 are 0.0)
Real1 || Real2        (1 if either Real1 or Real2 are not 0.0)
!  Real1              (1 if Int1 is 0)
Array1 && Array2      (Element-by-element logical operation)
Array1 || Array2      (Element-by-element logical operation)
!  Array1             (Element-by-element logical operation)
```

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