Setting up MATX-based Laboratory Experimental Systems

- A Pendulum System with Windows OS -

Masami Iwase†
Shingo Kojima‡
Teruyoshi Sadahiro†
Shoshiro Hatakeyama†

† Department of Robotics and Mechatronics, Tokyo Denki University, Tokyo, Japan.
‡ Graduate School of Computers and Systems Engineering, Tokyo Denki University, Saitama, Japan.

What is MaTX?

- MaTX is a programming language compatible with C, and equips functions for control analysis and control design.

In Tokyo Denki Univ. (TDU)

- **In the first grade**: (Programming Language)
  A simple matrix calculation

Education of TDU with MaTX
in the first grade

- A simple matrix calculation

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

- Bode diagram

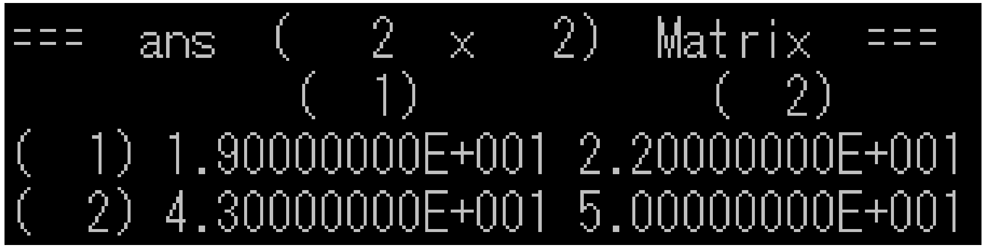
Bode diagram of $G(s) = \dfrac{1}{1 + 0.01s}$

MaTX is compatible with C, so a student is easy to use MaTX if the student has used C.

Education of TDU with MaTX in the first grade

- A simple matrix calculation

```
Func void main(){
        Matrix A,B;
        A = [[1,2][3,4]];
        B = [[5,6][7,8]];
        print A*B;}
```



```
===  ans  (   2  x   2)  Matrix  ===
              (  1)          (  2)
(  1) 1.90000000E+001 2.20000000E+001
(  2) 4.30000000E+001 5.00000000E+001
```

Fig.1 Output of matrix calculation

- Bode diagram

Bode diagram of $G(s) = \dfrac{1}{1+0.01s}$

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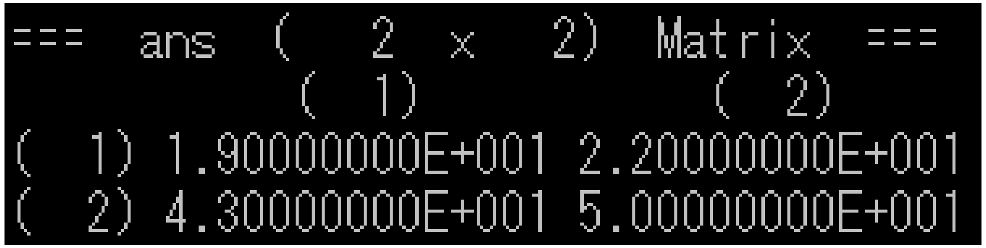


```
===   ans   (   2  x   2)  Matrix   ===
                (   1)            (  2)
(   1) 1.90000000E+001 2.20000000E+001
(   2) 4.30000000E+001 5.00000000E+001
```

- Bode diagram

```
Func void main(){
   Polynomial s;
   Rational G;
   s=Polynomial("s");
   G=1/(1+0.01*s);
   bode_plot_tfm([G]);}
```

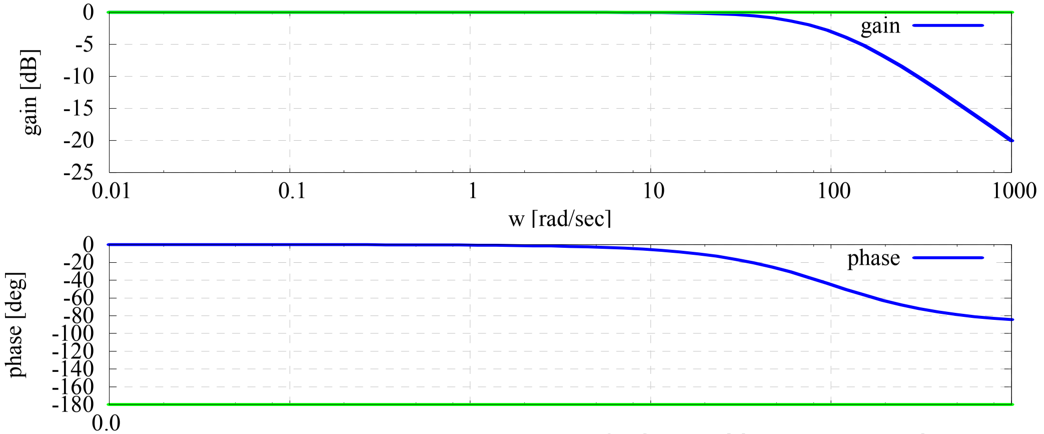


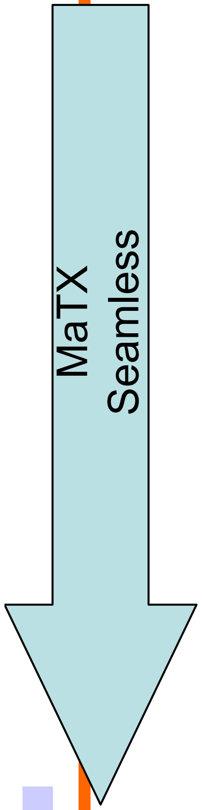
Fig.2 Bode diagram of G=1/(1+0.01s)

MaTX is compatible with C, so a student is easy to use MaTX if the student has used C.

Education of TDU with MaTX

- MaTX is a programming language compatible with C, and equips functions for analysis and design.

In Tokyo Denki Univ. (TDU)



•In the first grade: (Programming Language)
A simple matrix calculation
•In the second grade: (Tools for Control system analyses)
Bode Plot, Impulse･step response, Julia set
•In the third grade: (Tools for Control system design, Simulation)
A easy control method simulation
and implementation, analysis
•In the fourth grade:
(Experimental environment)
A complicate control method
simulation and implementation,
undergraduate researches

Example: Swing-up of Furuta pendulum by SDRE control

Tab.1 Parameters in the Furuta pendulum

Mass of pendulum	m_p	0.065 [kg]
Distance from the pivot to CG of pendulum	ℓ	0.123 [m]
Length of arm	r	0.215 [m]
Moment of Inertia of pendulum	J	0.0001554 [kgm^2]
Moment of Inertia of arm	J_a	0.0669 [kgm^2]
Gravity acceleration	g	9.81 [m/s^2]
Viscus friction coefficient of the pivot	c_p	2.102×10^{-4}[Nms/rad]
Viscus friction coefficient of the motor	c_a	0.0926779 [Nms/rad]

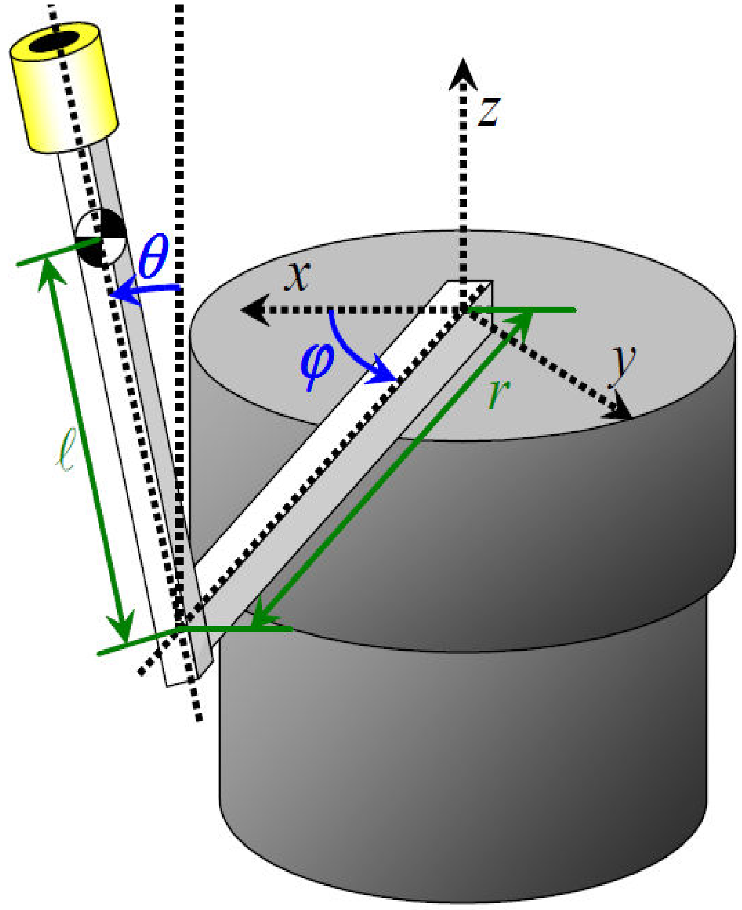


Fig.3 Schematic figure of the Furuta pendulum

Example: Swing-up of Furuta pendulum by SDRE control

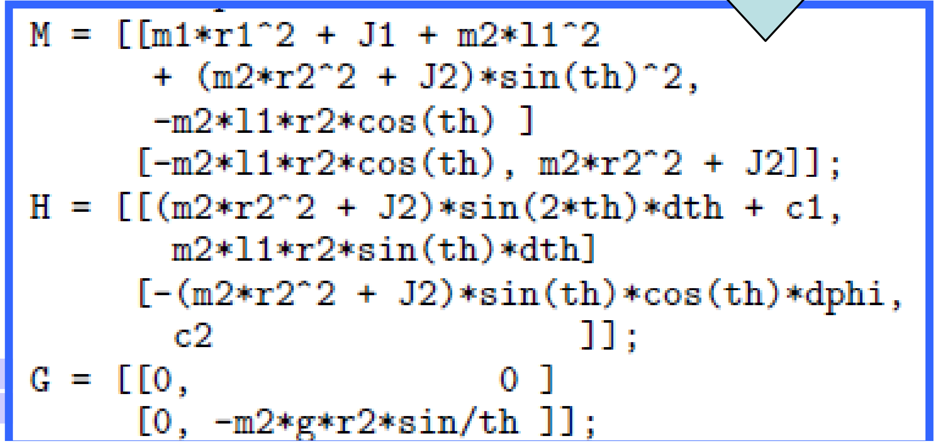
EOM of the Furuta pendulum

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)q = \tau\ , \quad q = \begin{bmatrix}\varphi \\ \theta\end{bmatrix} \qquad (1)$$

$$M(q) = \begin{bmatrix} J_a + m_p r^2 + (J + m_p l^2)\sin^2\theta & -m_p r l \cos\theta \\ -m_p r l \cos\theta & J + m_p l^2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} (J + m_p l^2)\dot{\theta}\sin(2\theta) + c_a & m_p r l \dot{\theta}\sin\theta \\ -(J + m_p l^2)\dot{\varphi}\sin\theta\cos\theta & c_p \end{bmatrix}$$

$$G(q) = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-m_p l g \sin\theta}{\theta} \end{bmatrix}$$

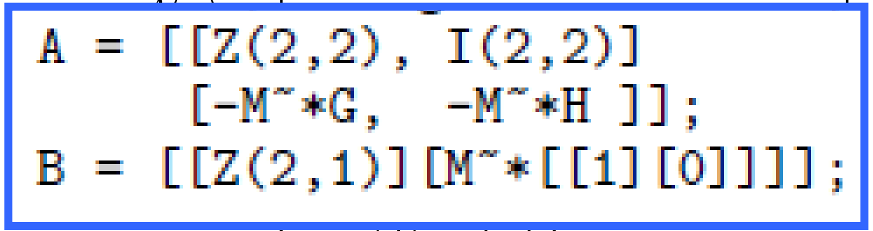


```
M = [[m1*r1^2 + J1 + m2*l1^2
      + (m2*r2^2 + J2)*sin(th)^2,
      -m2*l1*r2*cos(th) ]
     [-m2*l1*r2*cos(th), m2*r2^2 + J2]];
H = [[(m2*r2^2 + J2)*sin(2*th)*dth + c1,
      m2*l1*r2*sin(th)*dth]
     [-(m2*r2^2 + J2)*sin(th)*cos(th)*dphi,
      c2                         ]];
G = [[0,                    0 ]
     [0, -m2*g*r2*sin/th ]];
```

SDC state-space representation

$$\dot{x} = A(x)x + B(x)u\ , \quad x = \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix}^T \qquad (2)$$

From (1)

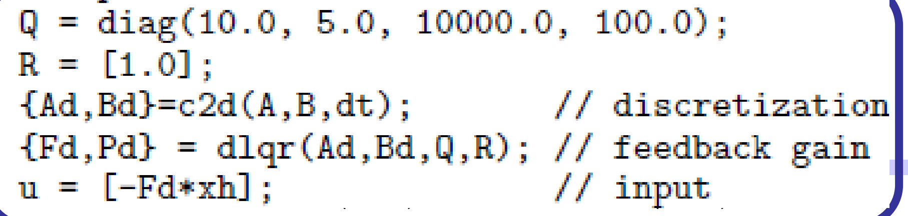


```
A = [[Z(2,2), I(2,2)]
     [-M~*G,   -M~*H ]];
B = [[Z(2,1)][M~*[[1][0]]]];
```

At *k*-th sampling period,
we freeze the state x as x_k .

$$\dot{x} = A(x_k)x + B(x_k)u$$

Discretization



```
Q = diag(10.0, 5.0, 10000.0, 100.0);
R = [1.0];
{Ad,Bd}=c2d(A,B,dt);        // discretization
{Fd,Pd} = dlqr(Ad,Bd,Q,R); // feedback gain
u = [-Fd*xh];               // input
```

Example: Swing-up of Furuta pendulum by SDRE control

Controller program

```
// SDC representation
M = [[m1*r1^2 + J1 + m2*l1^2
      + (m2*r2^2 + J2)*sin(th)^2,
      -m2*l1*r2*cos(th) ]
     [-m2*l1*r2*cos(th), m2*r2^2 + J2]];
H = [[(m2*r2^2 + J2)*sin(2*th)*dth + c1,
       m2*l1*r2*sin(th)*dth]
     [-(m2*r2^2 + J2)*sin(th)*cos(th)*dphi,
       c2                     ]];
G = [[0,                 0 ]
     [0, -m2*g*r2*sin/th ]];
A = [[Z(2,2), I(2,2)]
     [-M~*G,   -M~*H ]];
B = [[Z(2,1)][M~*[[1][0]]]];

// Optimal control
Q = diag(10.0, 5.0, 10000.0, 100.0);
R = [1.0];
{Ad,Bd}=c2d(A,B,dt);       // discretization
{Fd,Pd} = dlqr(Ad,Bd,Q,R); // feedback gain
u = [-Fd*xh];              // input
```

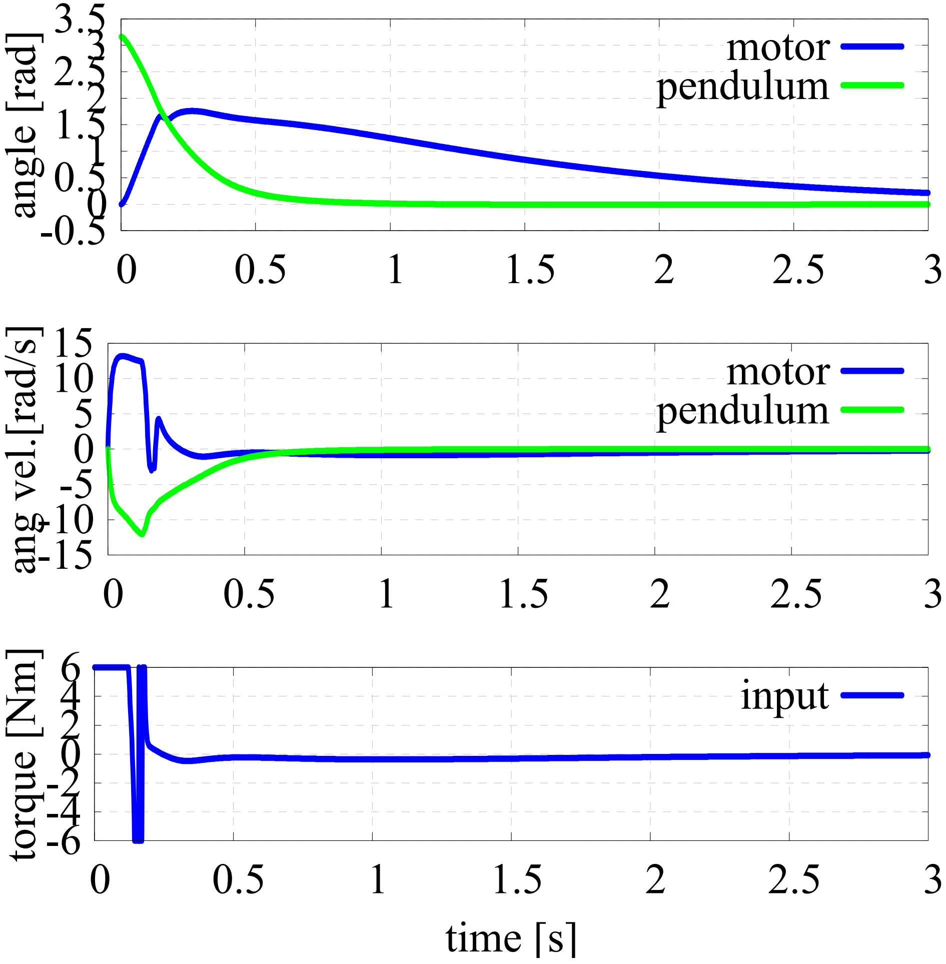
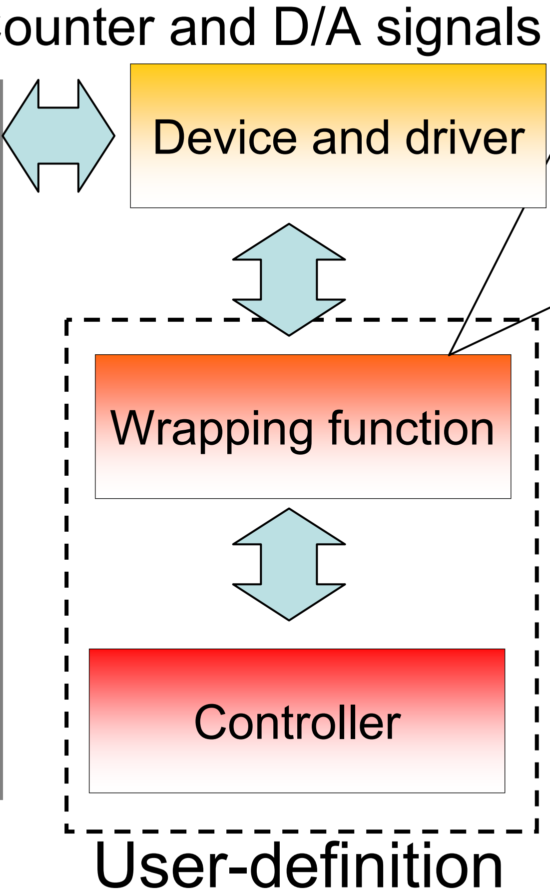


Fig.4 Simulation result

Experimental Furuta Pendulum System



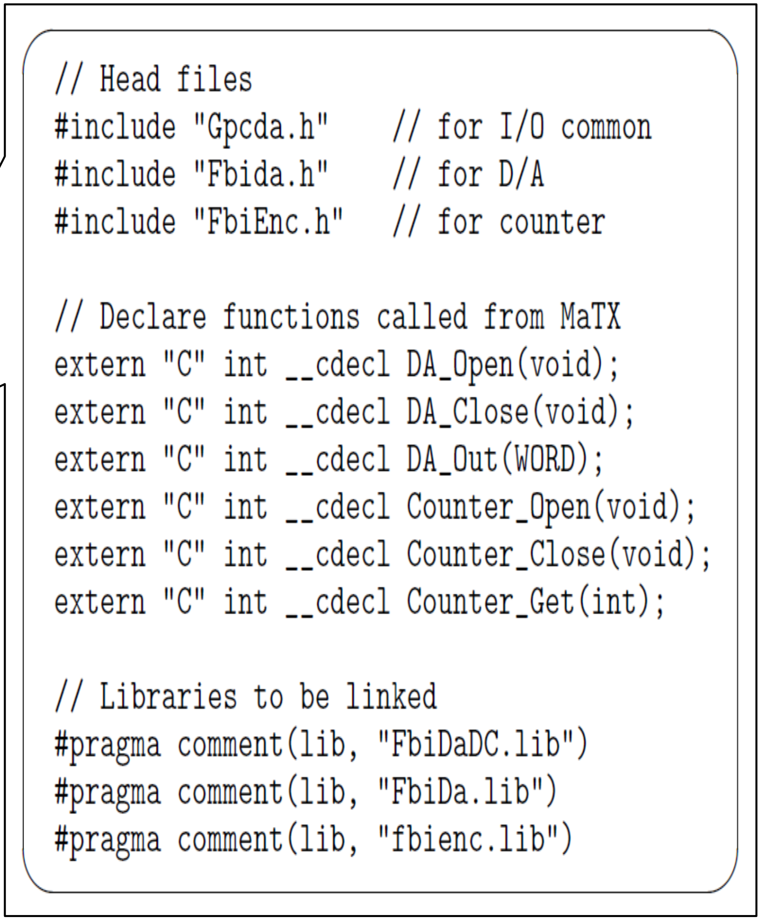
Counter and D/A signals

Device and driver

Wrapping function

Controller

User-definition

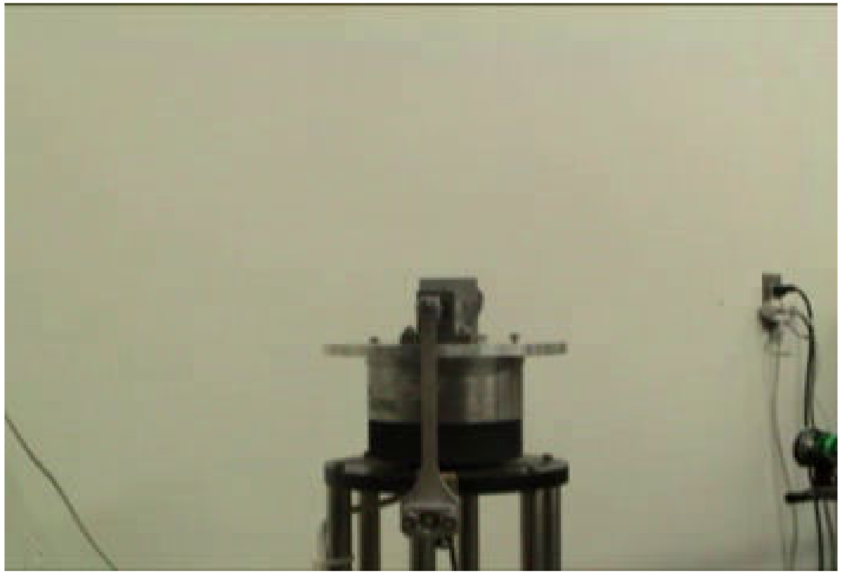


```
// Head files
#include "Gpcda.h"     // for I/O common
#include "Fbida.h"     // for D/A
#include "FbiEnc.h"    // for counter

// Declare functions called from MaTX
extern "C" int __cdecl DA_Open(void);
extern "C" int __cdecl DA_Close(void);
extern "C" int __cdecl DA_Out(WORD);
extern "C" int __cdecl Counter_Open(void);
extern "C" int __cdecl Counter_Close(void);
extern "C" int __cdecl Counter_Get(int);

// Libraries to be linked
#pragma comment(lib, "FbiDaDC.lib")
#pragma comment(lib, "FbiDa.lib")
#pragma comment(lib, "fbienc.lib")
```

Fig.5 A part of wrapping function

Therefore what we need prepare from software aspect is only two programs, MaTX program and wrapping functions.

Experimental Furuta Pendulum System



Experimental result

MaTX can use the same algorithm through simulation to experiment.

```
// Get information from sensors
   ≀
// SDC representation
M = [[m1*r1^2 + J1 + m2*l1^2
      + (m2*r2^2 + J2)*sin(th)^2,
      -m2*l1*r2*cos(th) ]
     [-m2*l1*r2*cos(th), m2*r2^2 + J2]];
H = [[(m2*r2^2 + J2)*sin(2*th)*dth + c1,
       m2*l1*r2*sin(th)*dth]
     [-(m2*r2^2 + J2)*sin(th)*cos(th)*dphi,
       c2                     ]];
G = [[0,              0 ]
     [0, -m2*g*r2*sin/th ]];
A = [[Z(2,2), I(2,2)]
     [-M~*G,  -M~*H ]];
B = [[Z(2,1)][M~*[[1][0]]]];

// Optimal control
Q = diag(10.0, 5.0, 10000.0, 100.0);
R = [1.0];
{Ad,Bd}=c2d(A,B,dt);        // discretizatio
{Fd,Pd} = dlqr(Ad,Bd,Q,R); // feedback gain
u = [-Fd*xh];               // input
// Output input signal
   ≀
```

Fig.6 A part of controller

Jitter Variation Test

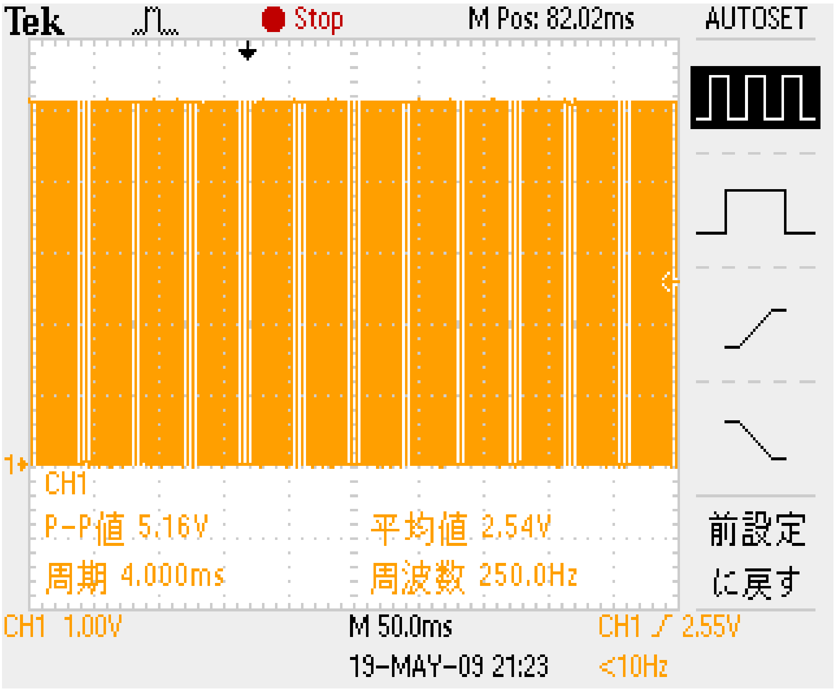


Fig.7 A pulse pattern generated by RT-MaTX with 4.0[msec] sampling interval

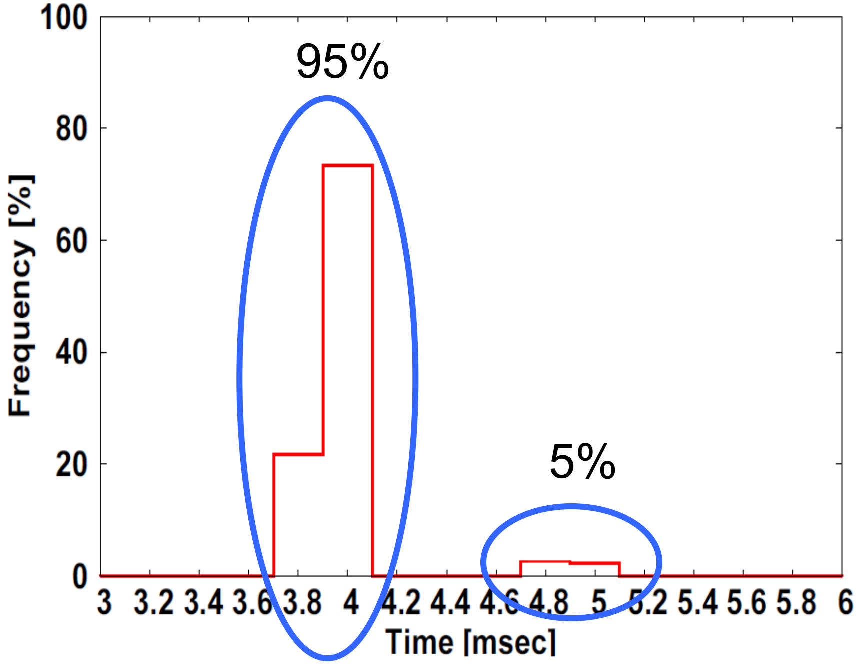


Fig.8 The histogram of jitter variation

MaTX is not proper for real products because of these undesirable longer sampling period.

But, it may use for education especially for implementing control algorithms on experimental systems.

Advantages of MaTX(1)

- MaTX can be seamless educational tools.
- MaTX can easily calculate matrix.
  - Comparing with Native C code
- When implemented algorithm, MaTX can utilize algorithm used in simulation.
  - Comparing with Native code oriented to hardware. (For example C)
- MaTX is free software.
  - Comparing with commercial softwares, MATLAB, LabView.
    Expensive! to buy and update them!

Advantages of MaTX(2)

- MaTX can be used Anytime, Anyway.
  - If MATLAB is used under floating license, the place where MATLAB can be used is limited.
- MaTX allows users to intuitively describe equations like multiplication of matrices in the manner of C language.
  - Ex. MaTX accepts A*B even if A and B are matrices. Similar with MATLAB commands.
- For education aspect, it is good occasion for uses to pay attention to internal algorithms.
  - For example how to deal with algebraic loops in a block diagram, transformation during different types.

Disadvantages of MaTX

We hope … (it’s a kind of disadvantages)

- Basically real-timeness depends on Windows OS.

  ↓

  Windows OS is not REALTIME OS.

  ↓

  We need MaTX running on some real-time OS or faster computing systems in which TICKs (like task-switching timing of OS) can be shorter.
- To use MaTX for industrial uses.

  For example, embedded systems.
- Debugger

Conclusion

Students sides

- I have understood the importance of MATX as the grade advanced, by education.
- An advantages and a disadvantages of MATX are more seen by MATLAB etc. also concurrently learn.

Teaching sides

- Introduced an seamless education based on MaTX at Tokyo Denki University
  - As an exmaple, Swinging-up a Furuta pendulum by a SDRE control was demonstrated as a sample case of the proposed experimental.

Thank you for your attention